Content & Form in Mathematics Presenting and Capturing Mathematics for the Web in *MathML*

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- Join the MathML Association (`http://mathml-association.org/`)

MathML: Mathematical Markup Language

> *MathML* is an XML application for describing mathematical notation and capturing both its structure and content. The goal of *MathML* is to enable mathematics to be served, received, and processed on the World Wide Web, just as HTML has enabled this functionality for text.
>
> *from the MathML2 Recommendation*

Representation of Formulae as Expression Trees

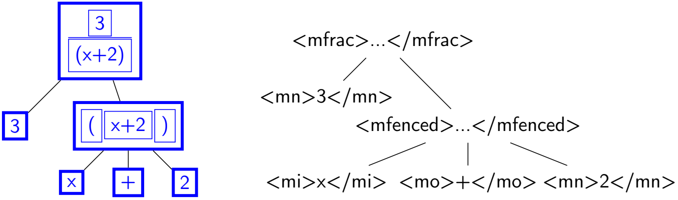
- Mathematical Expressions are build up as expression trees
  - of layout schemata in Presentation-*MathML*
  - of functional subexpressions in Content-*MathML*
- Example: $\frac{3}{(x+2)}$

```
<mfrac>
  <mn>3</mn>
  <mfenced>
      <mi>x</mi>
      <mo>+</mo>
      <mn>2</mn>
   </mfenced>
</mfrac>
```

```
<apply>
  <divide/>
  <cn>3</cn>
  <apply>
    <plus/>
    <ci>x</ci>
    <cn>2</cn>
  </apply>
</apply>
```

Layout Schemata and the *MathML* Box model

- Presentation MathML represents the visual appearance of a formula in a tree of layout primitives
- **Example 0.1 (Presentation MathML for $3/(x+2)$).**



P-*MathML* Token Elements

- Tokens Elements directly contain character data (the only way to include it)
  Attributes: fontweight, fontfamily and fontstyle, color...
- Identifiers: **<mi>**... **</mi>** (∼ variables, italicized)
- Numbers: **<mn>**... **</mn>** (numbers)
- Operators: **<mo>**... **</mo>** (constants, functions, upright)
- Operator display is often ideosyncratic (Operator Dictionaries for defaults)
  - Examples: spacing, *-scripts in sums and limits, stretchy integrals,...
  - Attributes: lspace, rspace, stretchy, and movablelimits.
  - Operators include delimiter characters like
    - parentheses (which stretch),
    - punctuation (which has uneven spacing around it) and
    - accents (which also stretch).

MathML Symbols in UniCode

- Problem: Mathematical formula use lots of non-`ASCII` symbols (not on your keyboard)
- Math Symbols: α, β, …Θ, $\int$, ⊎, $\pm$, ∞, $\mathbb{N}$, $\mathbb{R}$, … (+ ca. 5000 more)
- Recap: The UniCode standard collects all characters of all languages in the world. (100 000 so far)
- Idea: Math is a language, use UniCode for its characters.
- Recap: Each UniCode character is identified by an unambiguous name and an integer number called its code point (a number $\leq$ 1 100 000)
- **Example 0.2 (Some Math Symbols).**
  - The integral symbol $\int$ has the number U+8747 and the name `INTEGRAL`
  - The universal quantifier ∀ has the number U+8704 and the name `FOR ALL`
  - The letter θ has number U+952 and the name `GREEK SMALL LETTER THETA`

For *MathML*: UniCode letters can be used in HTML directly (and in *MathML*). Encode them via their code point as θ (decimal) or θ (hex).

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- Problem: What is the difference (in meaning) between $3(a+b)$, $c(a+b)$, and $f(a+b)$?

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  - *three a plus b* vs. *c times a plus b* vs.
  - *f applied to the sum of a and b* or *f of a plus b*

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- *MathML* introduces "invisible" (non-marking) characters for this:

U+2061	FUNCTION APPLICATION	character showing function application in presentation tagging
U+2062	INVISIBLE TIMES	marks multiplication when it is understood without a mark
U+2063	INVISIBLE SEPARATOR	used as a separator, e.g., in indices
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- **Example 0.3.** Encode $f(a+b)$ as <mrow>fࠍ(a+b)</mrow>

General Layout Schemata

- horizontal row: <**mrow**>child1 ... </**mrow**> (alignment and grouping)
- fraction: <**mfrac**>numerator denominator </**mfrac**>
  Attribute: linethickness (set to 0 for binomial coefficients)
- Radicals: <**msqrt**>child1 ... </**msqrt**> and

  <**mroot**>base index</**mroot**>
- grouping with parenthesis: <**mfenced**>child ... </**mfenced**>
  Attributes: open="(" and close="]" to specify parentheses
- grouping and style: <**mstyle**>child ... </**mstyle**> (pre-set attributes)

First Practical Markup Challenge (aka. Practice Example)

- We will jointly practice with concrete examples, here $x^2 + 4x + 4 = 0$
- General Workflow: write, test, repeat until done.
  - bring out your favorite text editor. (it really does not matter which one)
  - prepare a HTML5 file test.html

```
<html>
  <body>
    testing a polynomial:
    <math displaystyle="true"> ...</math>
  </body>
</html>
```

  - have a look at it in `FireFox`
  - replace the <math> element by your markup for $x^2 + 4x + 4 = 0$
  - have a look at it in `FireFox` again (does it look right)

Example: $x^2 + 4x + 4 = 0$

just presentation	some structure
``<mrow> <msup> <mi>x</mi> <mn>2</mn> </msup> <mo>+</mo> <mn>4</mn> <mi>x</mi> <mo>+</mo> <mn>4</mn> <mo>=</mo> <mn>0</mn> </mrow>```	```<mrow> <mrow> <msup> <mi>x</mi> <mn>2</mn> </msup> <mo>+</mo> <mrow> <mn>4</mn> <mi>x</mi> </mrow> <mo>+</mo> <mn>4</mn> </mrow> <mo>=</mo> <mn>0</mn> </mrow>```

Example: Grouping Arguments by mfenced

$f(x+y)$	$f(x+y)$
`` <mrow> <mi>f</mi> <mfenced> <mrow> <mi>x</mi> <mo>+</mo> <mi>y</mi> </mrow> </mfenced> </mrow> ```	``` <mrow> <mi>f</mi> <mfenced> <mstyle color='#ff0000'> <mrow> <mi>x</mi> <mo>+</mo> <mi>y</mi> </mrow> </mstyle> </mfenced> </mrow> ```

Example: <mfrac> and <mroot>

	$\sqrt[3]{1 - \frac{x}{2}}$

Example: <mfrac> and <mroot>

`` <mroot> <mrow> <mn>1</mn> <mo>−</mo> <mfrac> <mi>x</mi> <mn>2</mn> </mfrac> </mrow> <mn>3</mn> </mroot> ```	$\sqrt[3]{1-\frac{x}{2}}$

Example: The quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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```
<mrow>
  <mi>x</mi>
  <mo>=</mo>
  <mfrac>
    <mrow>
      <mrow><mo>−</mo><mi>b</mi></mrow>
      <mo>&plusmn;</mo>
      <msqrt>
        <mrow>
          <msup><mi>b</mi><mn>2</mn></msup>
          <mo>−</mo>
          <mrow><mn>4</mn><mi>a</mi><mi>c</mi></mrow>
        </mrow>
      </msqrt>
    </mrow>
    <mrow><mn>2</mn><mo>&InvisibleTimes;</mo><mi>a</mi></mrow>
  </mfrac>
</mrow>
```

Script Schemata

- Indices: G^1, H_5, R^i_j...
  - Super: <**msup**>base script </**msup**>
  - Subs: <**msub**>base script </**msub**>
  - Both: <**msubsup**>base superscript subscript</**msub**> (vertical alignment!)
- Bars and Arrows: $\overline{X}$, $\underbrace{Y}$,lue $\underbrace{\overline{Z}}$,...
  - Under: <**munder**>base script</**munder**>
  - Over: <**mover**>base script</**mover**>
  - Both: <**munderover**>base underscript overscript </**munderover**>
- Tensor-like: use <none/> for missing scripts

```
<mmultiscripts>
   base (sub sup)* [<mprescripts/> (psub psup)*]
</mmultiscripts>
```

msub + msup vs. msubsup

msub + msup	msubsup
`<msup>` `  <msub>` `    <mi>x</mi>` `    <mn>1</mn>` `  </msub>` `  <mi>α</mi>` `</msup>`	`<msubsup>` `  <mi>x</mi>` `  <mn>1</mn>` `  <mi>α</mi>` `</msubsup>`
${x_1}^{\alpha}$	x_1^{α}

Example: Movable Limits on Sums

- **Example 0.4.** $\displaystyle\sum_{i=1}^{\infty} x^i + \textstyle\sum_{i=1}^{\infty} x^i$

```
<mrow>
  <mstyle displaystyle='true'>
    <munderover>
      <mo>&sum;</mo>
      <mrow><mi>i</mi><mo>=</mo><mn>1</mn></mrow>
      <mi>&infty;</mi>
    </munderover>
    <msup><mi>x</mi><mi>i</mi></msup>
  </mstyle>
  <mo>+</mo>
  <mstyle displaystyle='false'>
    <munderover>
      <mo>&sum;</mo>
      <mrow><mi>i</mi><mo>=</mo><mn>1</mn></mrow>
      <mi>&infty;</mi>
    </munderover>
    <msup><mi>x</mi><mi>i</mi></msup>
  </mstyle>
</mrow>
```

Content Mathml: Expression Trees in Prefix Notation I

- Prefix Notation saves parentheses (so does postfix, BTW)

$(x-y)/2$	$x-(y/2)$
<**apply**> <**divide**/> <**apply**> <**minus**/> <**ci**>x</**ci**> <**ci**>y</**ci**> </**apply**> <**cn**>2</**cn**> </**apply**>	<**apply**> <**minus**/> <**ci**>x</**ci**> <**apply**> <**divide**/> <**ci**>y</**ci**> <**cn**>2</**cn**> </**apply**> </**apply**>

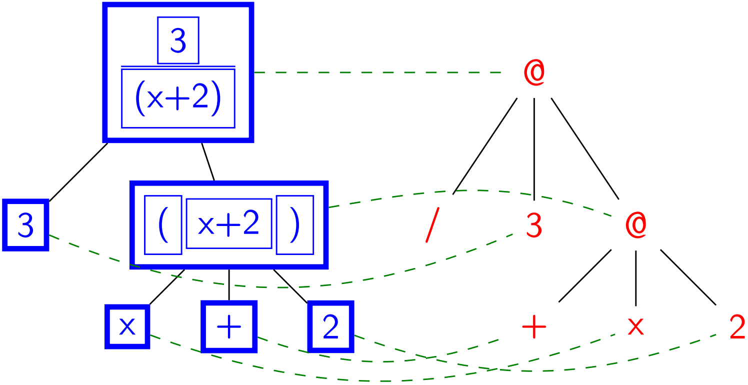
Function Application: <**apply**>function arg1 ... argn </**apply**>

Content Mathml: Expression Trees in Prefix Notation II

- Operators and Functions: ∼ 100 empty elements **<sin/>**, **<plus/>**, **<eq/>**, **<compose/>**,...
- Token elements: ci, cn (identifiers and numbers)
- Extra Operators: <**csymbol** cd="...">...</**csymbol**>

Parallel Markup e.g. in *MathML* I

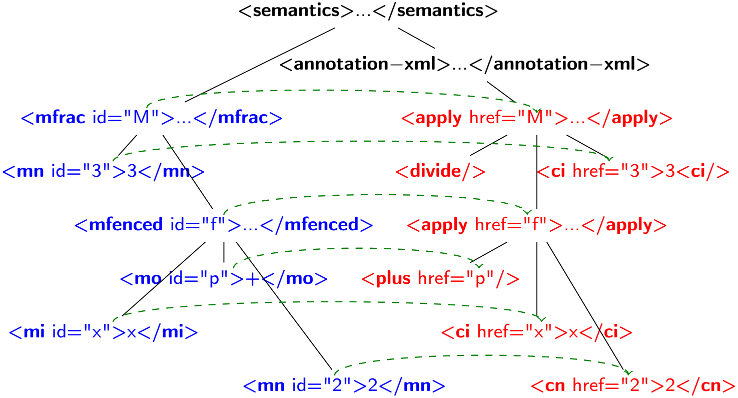
- Idea: Combine the presentation and content markup and cross-reference



- use e.g. for semantic copy and paste. (click o3n presentation, follow link and copy content)

Parallel Markup e.g. in *MathML* II

- Concrete Realization in *MathML*: semantics element with presentation as first child and content in annotation−xml child



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Content Mathml: Expression Trees in Prefix Notation II

- Operators and Functions: $\sim$ 100 empty elements <**sin**/>, <**plus**/>, <**eq**/>, <**compose**/>,...
- Token elements: ci, cn (identifiers and numbers)
- Extra Operators: <**csymbol** cd="...">...</**csymbol**>

Examples of Content Math

Expression	Markup
`<apply>` `  <plus/>` `  <apply><sin/><ci>x</ci></apply>` `  <cn>9</cn>` `</apply>`	$\sin(x) + 9$

Examples of Content Math

`<apply><eq/><ci>x</ci><cn>1</cn></apply>`	$x = 1$

Examples of Content Math

<apply><eq/> <bind><int/> <bvar><ci>x</ci></bvar> <apply><sin/><ci>x</ci></apply> </bind> <cos/> </apply>	$\int \sin(x)dx = \cos$

Examples of Content Math

<bind> <apply> <csymbol cd=''calculus1''>defint</csymbol> <cn>0</cn> <csymbol cd=''nums1''>infinity</csymbol> </apply> <bvar><ci>x</ci></bvar> <apply><sin/><ci>x</ci></apply> </bind>	$\int_0^\infty \sin(x) dx$

Examples of Content Math

``<bind>``` ``` <apply><sum/>``` ``` <cn>0</cn><ci>&infty;</ci>``` ``` </apply>``` ``` <bvar><ci>n</ci></bvar>``` ``` <apply><power/><ci>x</ci><ci>n</ci></apply>``` ```</bind>```	$\sum_0^\infty x^n$

Examples of Content Math

`<bind>` `  <set/>` `  <bvar><ci>x</ci></bvar>` `  <bvar><ci>y</ci></bvar>` `  <apply><and/>` `    <apply><lt/>` `      <ci>0</ci><ci>x</ci><ci>1</ci>` `    </apply>` `   <apply><leq/>` `     <ci>3</ci><ci>y</ci><ci>10</ci>` `   </apply>` `</bind>`	$\left\{x, y \,\middle\|\, \begin{array}{l} 0 < x < 1, \\ 3 \leq y \leq 10 \end{array}\right\}$

Examples of Content Math

Expression	Markup
<apply><eq/> <bind><set/> <bvar><ci>x</ci></bvar> <apply><geq/> <ci>x</ci><cn>0</cn> </apply> </bind> <apply> <cointerval/> <cn>0</cn> <cn>&infty;</cn> </interval> </apply>	$\{x \vert x \geq 0\} = [0, \infty)$

Examples of Content Math

```
<apply><eq/>
  <apply><times/>
    <apply><vector/>
        <cn>1</cn><cn>2</cn>
    </apply>
    <apply><matrix/>
      <apply><matrixrow/>
           <cn>0</cn><cn>1</cn>
      </apply>
      <apply><matrixrow/>
           <cn>1</cn><cn>0</cn>
       </apply>
    </apply>
    <apply>
      <transpose/>
      <apply><vector/>
          <cn>2</cn><cn>1</cn>
    </apply>
  </apply>
</apply>
```

$$(1,2) \times \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (2,1)^t$$

From Presentation to Content?

- Problem: Presentation Markup $\leftrightarrow$ Content Markup
  - many presentation for one concept (e.g. binomial coeff. $\binom{n}{k}$ vs. C_k^n vs. C_n^k)
  - many concepts for one presentation (e.g. m^3 is m cubed, cubic meter, upper index, footnote,...)
  - grouping is left implicit, invisible operators (e.g. $3a^2+6ab+b^2$)
  - disambiguation by context (e.g. $\lambda X_\alpha.X =_\alpha \lambda Y_\alpha.Y$)
  - notation is introduced and used on the fly.
- Content Recovery is a heuristic context/author-dependent process
  - There is little hope we can do it fully automatically in principle (AI-hard!)
  - for limited domains we can do a good job (e.g. in Mathematica 4)

Added-value services with Math Content

- cut and paste (cut output from web search engine and paste into CAS)
- automatically proof checking formal argumentations (bridge verification?)
- math explanation (e.g. specialize a proof to a simpler special case)
- semantical search for mathematical concepts (rather than keywords)
- data mining for representation theorems (find unnoticed groups out there)
- classification (given a concrete math structure, is there a general theory?)
- personalized notation (implication as $\rightarrow$ vs. $\supset$, or Ricci as $\frac{1}{2}\mathcal{R}^{ij}$ vs. $2\mathcal{R}^{ij}$)
- user-adapted documents (ActiveMath, Course Capsules)

The arXMLiv Project: arXiv to semantic XML

- Idea: Develop a large corpus of knowledge in HTML5
  - to get around the chicken-and-egg problem of MKM/GDML
  - corpus-linguistic methods for semantics recovery (linguists interested)

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- **Definition 0.6 (The arXMLiv Project).** (http://arxmliv.kwarc.info)
  - use Bruce Miller's LaTeXML to transform to HTML5
  - extend to LaTeXML daemon (RESTful web service) (http://latexml.mathweb.org)
  - we have an automated, distributed build system (ca. 4 CPU-years)
  - create ca. 13K LaTeXML binding files (100 done $\widehat{=}$ 80% coverage)
  - use MathWebSearch to index XML version (realistic search corpus)

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  - use MathWebSearch to index XML version (realistic search corpus)
- More semantic information will enable more added-value services, e.g.
  - filter hits by model assumptions (expanding, stationary, or contracting universe)
  - use linguistic techniques to add the necessary semantics

Semantics Extraction, e.g. Quantity Expressions

- Idea: Find characteristic patterns in mathematical documents.
- **Example 0.7.** Quantity expressions, e.g.
  - *five seconds*
  - $1.0 \cdot 10^{17} W/cm^2$ (Watt per square cm)
  - $0.6 M_\odot$ (solar masses)
  - $0.53 \pm 0.01 eV$ (range)
- Problem: Ambiguity
  - *GHz* is could be *gigahertz*, but could also denote *GauÃ§ · Hertz*.
  - *Pa* has two possible meanings – *petayear* and *Pascal*.
- Problem: Context Dependency
  - $3m/s$ vs. $E = mc^2$. (n is “meter” or “mass”)
- Applications: that make use of the semantics
  - screen readers for the vision-impaired: read *3m/s* as *three meters per second* instead of *three m slash s*.
  - physical search engines: search for *3m/s, find 10.8 km/h* or *18 037 furlongs per fortnight*
  - document localization: show a recipe with *8 oz of butter* as *225 g of butter*.

Example Services

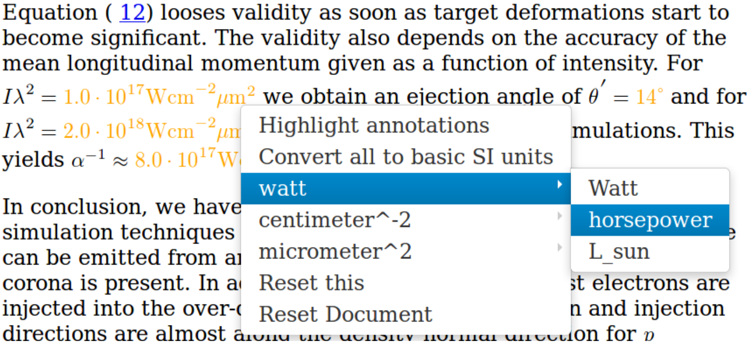
- **Example 0.8 (Highlighting Quantity Expressions).**

$$\tan\theta' = \frac{\sqrt{1+\alpha I\lambda^2}-1}{\sqrt{\alpha I\lambda^2}}\tan\theta\,. \quad (12)$$

Equation (12) looses validity as soon as target deformations start to become significant. The validity also depends on the accuracy of the mean longitudinal momentum given as a function of intensity. For $I\lambda^2 = 1.0\cdot 10^{17}\mathrm{Wcm}^{-2}\mu\mathrm{m}^2$ we obtain an ejection angle of $\theta' = 14^\circ$ and for $I\lambda^2 = 2.0\cdot 10^{18}\mathrm{Wcm}^{-2}\mu\mathrm{m}^2$ we obtain $\theta' = 17^\circ$ from the simulations. This yields $\alpha^{-1} \approx 8.0\cdot 10^{17}\mathrm{Wcm}^{-2}\mu\mathrm{m}^2$.

Example Services

- **Example 0.8 (Highlighting Quantity Expressions).**
- **Example 0.9 (In-Situ Conversion).** Chossing a target unit



Example Services

- **Example 0.8 (Highlighting Quantity Expressions).**
- **Example 0.9 (In-Situ Conversion).** Converting one occurrence

  Equation (12) looses validity as soon as target deformations start to become significant. The validity also depends on the accuracy of the mean longitudinal momentum given as a function of intensity. For $I\lambda^2 = 1.34 \cdot 10^{14} \cdot \text{horsepower} \cdot \text{centimeter}^{-2} \cdot \text{micrometer}^2$ we obtain an ejection angle of $\theta' = 14^\circ$ and for $I\lambda^2 = 2.0 \cdot 10^{18}\text{Wcm}^{-2}\mu\text{m}^2$ we obtain $\theta' = 17^\circ$ from the simulations. This yields $\alpha^{-1} \approx 8.0 \cdot 10^{17}\text{Wcm}^{-2}\mu\text{m}^2$.

Example Services

- **Example 0.8 (Highlighting Quantity Expressions).**
- **Example 0.9 (In-Situ Conversion).** Converting all occurrences

  Equation (12) looses validity as soon as target deformations start to become significant. The validity also depends on the accuracy of the mean longitudinal momentum given as a function of intensity. For $I\lambda^2 = 1.00 \cdot 10^9 \cdot \mathrm{m}^2 \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$ we obtain an ejection angle of $\theta' = 0.244 \cdot \mathrm{rad}$ and for $I\lambda^2 = 2.00 \cdot 10^{10} \cdot \mathrm{m}^2 \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$ we obtain $\theta' = 0.297 \cdot \mathrm{rad}$ from the simulations. This yields $\alpha^{-1} \approx 8.00 \cdot 10^9 \cdot \mathrm{m}^2 \cdot \mathrm{kg} \cdot \mathrm{s}^{-3}$.

References I