1 Radioactive Decay

Background In September 1991 the famous Iceman (Ötzi), a mummy from the Neolithic period of the Stone Age found in the ice of the Ötztal Alps (hence the name Ötzi) in Southern Tyrolia near the Austrian-Italian border, caused a scientific sensation.

Problem When did Ötzi approximately live and die if the ratio of carbon ${}_6C^{14}$ to carbon ${}_6C^{12}$ in this mummy is 52.5%?

Physical Information In the atmosphere and in living organisms, the ratio of radioactive ${}_6C^{14}$ (made radioactive byt cosmic rays) to ordinary ${}_6C^{12}$ is constant. When an organism dies, its absorption of ${}_6C^{14}$ by breathing and eating terminates. Hence one can estimate the age of a fossil by comparing the radioactive carbon ration in the fossil with that of the atmosphere. To do this one needs to know the half-life of ${}_6C^{14}$, which is 5715 years.

Solution Radioactive decay is governed by the ODE $y' = ky$. By separation and integration (where t is time and y_0 is the initial ratio of ${}_6C^{14}$ to ${}_6C^{12}$)

$$\frac{dy}{y} = k\,dt, \qquad \ln|y| = kt + c, \qquad y = y_0 e^{kt}$$

Next we use the half-life $H = 5715$ to determine k. When $t = H$, half of the original substance is still present, thus

$$y_0 e^{kH} = 0.5 y_0. \qquad e^{kH} = 0.5. \qquad k = \frac{\ln 0.5}{H} = -\frac{0.693}{5715} = -0.0001213.$$

Finally, we use the ration 52.5% for determining the time t when Ötzi died (actually was killed),

$$e^{kt} = e^{-0.0001213t} = 0.525, \qquad t = \frac{\ln 0.525}{-0.0001213} = 5312 \qquad \text{Answer: 5300 years ago}$$