Last Name: First Name:

Matriculation Number:

Exam
Symbolic Methods for AI

September 29. 2025

Please ignore the QR codes; do not write on them, they are for grading support

	To be used for grading, do not write here														
prob.	1.1	1.2	1.3	2.1	2.2	2.3	3.1	4.1	5.1	5.2	6.1	6.2	6.3	Sum	grade
total	6.5	9	2	4	4	3	8	8	4	3	2	2	5	60.5	
reached															

Organizational Information

Please read the following directions carefully and acknowledge them with your signature.

1. Please place your student ID card and a photo ID on the table for checking.
2. You can reach 60.5 points if you fully solve all problems. You will only need 55 points for a perfect score, i.e. 5.5 points are bonus points.
3. No resources or tools are allowed except for a pen.
4. In particular, you are not allowed any electronic devices – phones, smart watches, earbuds, smart rings, ...on your person. Put them into your backpack or your pockets.
5. You have 60 min (sharp) for the exam.
6. Write the solutions directly on the sheets, no other paper will be graded.
7. If you have to abort the exam for health reasons, your inability to sit the exam must be certified by an examination at the University Hospital. Please notify the exam proctors and have them give you the respective form.
8. Please make sure that your copy of the exam is complete (11 pages excluding cover sheet and organizational information pages) and has a clear print. **Do not forget to add your personal information on the cover sheet and to sign this declaration.**

Declaration: With my signature I certify having received the full exam document and having read the organizational information above.

Erlangen, September 29. 2025 .....
(signature)

Please consider the following guidelines to avoid losing points:

- If you continue an answer on another page, clearly give the problem number on the new page and a page reference on the old page.
- You can always ask for the translation or explanation of a non-technical word.
- If you do not want something to be graded, clearly cross it out. Adding a wrong statement to a correct solution may lead to deductions.
- The instructions "Give X", "List X" or similar mean that only X is needed. If you additionally justify your answer, we may or may not give you partial credit.
- The instruction "Assume X" means that X is information that your answer may use.
- The instruction "Model X as a Y" means that you have to describe X formally and exactly as an instance of Y using the definition of Y from the lecture.
- If you are uncertain how long or complex an answer should be, use the number of points as an indication: 1 point roughly corresponds to 1 minute.
- In all calculation questions, you have to simplify as much as reasonably possible without a calculator. For example, $\log 2$ or 3^7 should not be calculated, but $0.4 \cdot 0.3 \cdot 0.5 = 0.06$ should be.

1 Talking about Mathematics

Problem 1.1 (Talking about Sets)
Given the following sets

1. $A = \{a, b, c, d, e\}$
2. $B = \{d, f, h\}$
3. $C = \{d, f, g, i\}$

where the $a, \ldots, h, i$ are pairwise distinct.
Define each of the following operations on sets **in MathTalk** and apply it to the given sets:

1. Intersection (general formula in MathTalk) — 1 Points
2. $A \cap B =$ — 0.5 Points
3. Union (general formula in MathTalk) — 1 Points
4. $B \cup C =$ — 0.5 Points
5. Set difference (general formula in MathTalk) — 1 Points
6. $A \setminus B =$ — 0.5 Points
7. Cartesian product (general formula in MathTalk) — 1 Points

8. $B \times C =$

1 Points

Problem 1.2 (Properties of Relations)
Let R and S be relations on some given set A.

1. Prove or refute that

   If R and S are symmetric then $R \cap S$ is symmetric.

3 Points

2. Prove or refute that

   If R is reflexive then all subsets of R are reflexive.

3 Points

3. Prove or refute that

   If R is transitive then the converse relation R^{-1} is transitive.

3 Points

Problem 1.3 (Truths about Proofs) 2 Points

Which of the following statements about the truth of mathematical statements are correct?

- ☐ If I have a counterexample for A, then A is false.
- ☐ If I have an example for A, then A is true.
- ☐ If I do not have a proof for $\neg A$, then A is true.
- ☐ If I do not have a proof for A, then A is false.
- ☐ If I have a proof for A, then A is true.
- ☐ If I have a proof for $\neg A$, then A is false.

2 Standard ML

Problem 2.1 (SML Predicate for Reflexive Relations) 4 Points

Write an SML predicate (i.e. a function with result type `bool`) that given a list S and a list R of pairs whose components are in S, returns `true`, iff the relation R is reflexive on the set represented by the list S.

Hint: You can use a predicate `member: 'a -> 'a list -> bool` that checks whether the first argument is element of the second argument (a list) without defining it yourself.

Problem 2.2 (Mapping and Appending)

`mapcan` is a higher-order function that maps a list-valued function over a list and concatenates all the result lists into a single list.

1. What is the SML type of the function `mapcan` (explain in terms of the types of its arguments and

2 Points

results).

2. Implement `mapcan` in SML recursively.

2 Points

Problem 2.3 (Mapping and Appending) 3 Points

Can the function `mapcan` from above be written using `foldl`/`foldr`? If so give an implementation, if not, argue why not.

3 Graphs and Trees

Problem 3.1 (Graph and Trees)

1. Draw the parse tree of the expression string `(-X1+X3)*(-(-X2))` with respect to the grammar for arithmetic expressions presented in the SMAI course. 3 Points

2. Give an example of a different node labeled graph that is isomorphic to the parse tree above and write its mathematical representation in terms of concrete sets and functions. 3 Points

3. Can the graph underlying the parse tree be isomorphic to a cyclic graph? Justify why, or why not? 2 Points

4 Mathematical Structures

Problem 4.1 (Foobar Structures)
A structure $\langle S, v\rangle$, where S is a set, and $v : S \times S \to \mathbb{N}$ is a total function is called a **foobar**, iff

- $v(a,a) = 1$ for all $a \in S$ and
- for all $a, b \in S$ we have $a = b$, if $v(a,b) = v(b,a)$.

1. Provide a function v such that $\langle\{1\}, v\rangle$ is a foobar

2 Points

2. Provide a function v such that $\langle\{1,2,3\}, v\rangle$ is a foobar.

2 Points

3. Prove or refute:

   If $\langle S, v\rangle$ and $\langle S', v'\rangle$ are foobars, then $\langle S \times S', v''\rangle$ is a foobar where $v''((a,b),(c,d)) = v(a,c) + v'(b,d)$.

4 Points

5 Complexity Analysis

Problem 5.1 (Time complexity of an SML function)
Consider the the following SML function:

```
fun twist(nil) = nil
  | twist([hd1]) = [hd1]
  | twist(hd1::(hd2::tl)) = twist(hd1::tl)@twist(hd2::tl)
```

1. How many times is line 3 executed to compute `twist [5,6,2,9]`? 2 Points

2. What is the time complexity of `twist` ($\Theta(f)$) in terms of the length of the input list. Justify your answer! 2 Points

Problem 5.2 (Landau sets) 3 Points
Order the Landau sets below by specifying which ones are proper subsets and which ones are equal (e.g.: $\mathcal{O}(a) \subset \mathcal{O}(b) \subset \mathcal{O}(c) \equiv \mathcal{O}(d) \subset \mathcal{O}(e) \dots$)

$$\mathcal{O}(n^2);\ \mathcal{O}(n!);\ \mathcal{O}(|\sin n|);\ \mathcal{O}(n^n);\ \mathcal{O}(1);\ \mathcal{O}(2^n);\ \mathcal{O}(2n^2 + 2^{72})$$

6 Formal Languages and Grammars

Problem 6.1 (Grammars) 2 Points

Consider the following phrase structure grammar in BNF form:

$$\begin{array}{rcl} F & ::= & N(Ts) \mid F \wedge F \mid \forall N.F \\ T & ::= & N \mid N(Ts) \\ Ts & ::= & T \mid T, Ts \\ N & ::= & f \mid g \mid h \mid p \mid q \mid x \mid y \mid z \end{array}$$

Which of the following strings are derivable from F:

- ☐ $p(g(x, g(y)))$
- ☐ $f(p(x) \wedge q(y))$
- ☐ $p(f(x, y, z)) \wedge \forall x.p(y)$
- ☐ $\forall x, y.p(x, y)$

Problem 6.2 (Grammars) 2 Points

Which of the following are true statements:

- ☐ Every regular language has a context-free grammar.
- ☐ If every production rule of a grammar has a nonterminal symbol in its body, the language is empty.
- ☐ If the heads of all production rules in a grammar have exactly one symbol, the language is finite.
- ☐ If two grammars produce the same language, they have the same number of nonterminal symbols.

Problem 6.3 (Abstract and Concrete Grammars) 5 Points

Let L be a term language. Describe – in your own words the difference between an abstract grammar and a corresponding concrete grammar for L.

Hint: Think about brackets and grammar types/classes and the complexity difference between the two kinds.

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