Last Name: First Name:

Matriculation Number:

Exam
Logic-Based Natural Language Semantics

2025-08-12

Please ignore the QR codes; do not write on them, they are for grading support

	To be used for grading, do not write here													
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In the LBS course we award bonus points for the first student who reports a factual error in an old exam. (Please report spelling/formatting errors as well.)

1 Foundations

Problem 1.1 (PL^{nq})
Consider the following PL^{nq} formula:

$$\Theta := p(x, y) \wedge \neg p(f(x), y) \wedge p(f(x), x)$$

1. Specify the signature that makes Θ well-formed.

2 Points
Objective: apply first-order signature

Solution:

$$\Sigma_0^f = \{x, y\}$$
$$\Sigma_1^f = \{f\}$$
$$\Sigma_2^p = \{p\}$$

Grading:

AC1 looks like a signature
Points: +0.5
Feedback: The content is wrong but at least the result looks like a first-order signature

2. Given the first-order model $\langle \mathcal{D}, \mathcal{I} \rangle$ where

2 Points
Objective: understand first-order model
Objective: apply interpretation

$$\mathcal{D} = \{1, 2\}$$
$$\mathcal{I}(x) = 1$$
$$\mathcal{I}(y) = 2$$
$$\mathcal{I}(f) = \{(1,2), (2,2)\}$$
$$\mathcal{I}(p) = ???$$

List all values for $\mathcal{I}(p)$ (they are sets of pairs) such that Θ evaluates to T?

Solution: $\{(1,2), (2,1)\}$ and $\{(1,1), (1,2), (2,1)\}$

Grading:

AC1 only one of two possibilities listed
Points: 1.5

Problem 1.2

3 Points
Objective: apply beta reduction

Consider the following untyped λ-term:

$$(\lambda a.\lambda b.b\ a)\ x\ Q$$

Provide a value for Q such that the term β-reduces to $\lambda c.x$.

Solution: e.g. $Q = \lambda p.\lambda c.p$ or $Q = \lambda p.\lambda c.x$.

Problem 1.3

3 Points

Which of the following statements is true about the untyped lambda calculus Λ and the simply typed lambda calculus $\Lambda^{\to}$?

- [ ] Every term in Λ can be transformed into a well-typed term in $\Lambda^{\to}$ by adding types.
- [x] Every term in $\Lambda^{\to}$ can be transformed into a well-formed term in Λ by removing the types.
- [ ] β-reduction always terminates in Λ.
- [x] β-reduction always terminates in $\Lambda^{\to}$.
- [x] η-reduction always terminates in Λ.
- [x] η-reduction always terminates in $\Lambda^{\to}$.

Objective: understand STLC
Objective: understand UTLC
Objective: understand beta reduction
Objective: understand eta reduction

2 The Method of Fragments

Problem 2.1 (Method of Fragments)

4 Points

In the method of fragments, we specify the **A** *and* **B** *of subsets of natural language. The* **C** *is specified in a grammar, and the* **D** *is represented in a logic.*

Objective: understand fragment

Blank **A**:

- ○ subset relation
- ○ hierarchy
- ● syntax
- ○ β-reduction
- ○ tableau machine

Blank **B**:

- ○ lambda calculus
- ○ countability
- ○ superset relation
- ○ natural deduction calculus
- ● semantics

Blank **C**:

- ● syntax
- ○ semantics
- ○ superset
- ○ truth conditions

Blank **D**:

- ○ syntax
- ○ tableau machine
- ● semantics
- ○ β-expansion

Problem 2.2 (Adjective fragment)

Consider the following grammar with start symbol S on the left and the incomplete semantics construction on the right:

Objective: create grammar
Objective: create semantics construction

$$\begin{array}{llll} S1: & S \to NP, \text{is}, AP, & T1: & [X_{NP}, \text{is}, Y_{AP}]_S \rightsquigarrow Y'(X') \\ N1: & NP \to \text{Ali}, & T2: & [\text{Ali}]_{NP} \rightsquigarrow a \\ N2: & NP \to \text{Zahra}, & T3: & [\text{Zahra}]_{NP} \rightsquigarrow z \\ A1: & AP \to \text{happy}, & T4: & [\text{happy}]_{AP} \rightsquigarrow h \\ A2: & AP \to \text{bored}, & T5: & [\text{bored}]_{AP} \rightsquigarrow b \\ A3: & AP \to \text{not}, AP, & T6: & ??? \rightsquigarrow ??? \\ A4: & AP \to AP, \text{and}, AP & T7: & ??? \rightsquigarrow ??? \end{array}$$

1. Give an example sentence that is syntactically ambiguous in this grammar and demonstrate why it is ambiguous.

3 Points
Objective: understand ambiguity
Objective: create ambiguity

Solution: "Ali is not happy and bored" can be read as "Ali is (not happy) and bored" or "Ali is not (happy and bored)"

Grading:

AC1 contradiction instead of ambiguity
Points: 0.0
Feedback: This is a contradition, not an ambiguity; there is a difference.

AC2 no explanation
Points: -1
Feedback: The question requests a demonstration of ambiguity, this is missing.

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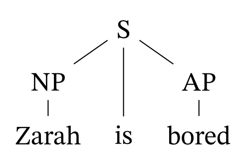
2. Apply the semantics construction to "Zahra is bored". Show the full derivation.

3 Points
Objective: apply semantics construction

Solution: The derivation in the grammar is $S \to NP\text{is}AP \to \text{Zarah is}AP \to \text{Zarah is bored}$, which leads to the syntax tree



Then we use rule $T1$ to turn the tree into $\text{bored}'(\text{Zarah}')$ and then further into $b(z)$ with $T2$ and $T3$.

Grading:

AC1 only parse tree
Points: 1
Feedback: The question asked for a derivation, you only gave a parse tree. There is a difference!

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3. Add the missing rules to the semantics construction.

4 Points
Objective: create semantics construction

Solution:
$T6: [\text{not}, X_{AP}]_{AP} \rightsquigarrow (\lambda y.\neg X'(y))$
$T7: [X_{AP}, \text{and}, Y_{AP}]_{AP} \rightsquigarrow (\lambda y.X'y \wedge Y'(y))$

Grading:

AC1 arguments treated as propositions, not predicates
Points: -1
Feedback: The arguments were treated as propositions, not as predicates as they should have been.

Problem 2.3 (Event semantics) 3 Points

How can the following sentence be represented in first-order logic using event semantics?
"John saw Mary in London at midnight"

Objective: apply
event semantics

Solution: $\exists e.type(e, see) \wedge agent(e, John) \wedge patient(e, Mary) \wedge location(e, London) \wedge time(e, midnight)$

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3 Inference in NLU

Problem 3.1 4 Points

Fill in the blanks according to letters

The tableau machine is a/an **A** *cognitive model for* **B** *that iteratively*

- *develops input sentences of a* **C** *D in a tableau*
- *chooses* **D** *tableau branches to act as models for D.*

Blank **A**:
- ○ ideological
- ☑ inference-based
- ○ neural
- ○ statistical

Blank **B**:
- ○ computation
- ○ theorem proving
- ☑ natural language understanding
- ○ natural language generation

Blank **C**:
- ☑ discourse
- ○ language
- ○ branch
- ○ grammar

Blank **C**:
- ○ closed
- ○ long
- ○ contradictory
- ☑ open

○ empty

Problem 3.2
Consider the following three-sentence discourse:

D1 *Peter is a dog or a cat.*

D2 *He is tall.*

D3 *He meows.*

We assume the following (background knowledge):

W1 Cats meow, but do not bark.

W2 Dogs bark, but do not meow.

Further more we assume that all occurrences of the pronoun *he* are resolved to *Peter*.
After the processing the first two sentences, the tableau machine is in the following state:

$$\begin{array}{c}
\boxed{dog(p) \vee cat(p)} \\
\begin{array}{c|c}
dog(p)^{\mathsf{T}} & cat(p)^{\mathsf{T}} \\
\boxed{tall(p)} &
\end{array}
\end{array}$$

The tableau machine has chosen the left branch in the tableau.

1. What happens when the tableau machine processes the third sentence in the discourse above?

4 Points
Objective: apply tableau machine

Solution:

1. The tableau machine adds the third sentence as the input logical form $\boxed{meow\,p}$ to the left branch
2. We can represent W2 as $\forall x.dog(x) \Rightarrow \neg meow(x)$ and add it to the left branch as $(\forall x.dog(x) \Rightarrow \neg meow(x))^{\mathsf{T}}$
3. chaining with W2 derives $\neg meow(p)^{\mathsf{T}}$ and then $meow(p)^{\mathsf{F}}$ and close the left branch.
4. Then we backtrack and add D2 to the right branch, saturate, and then add add D3 to the right branch and saturate.
5. The tableau has the following form now:

$$\begin{array}{c}
\boxed{dog(p) \vee cat(p)} \\
\begin{array}{c|c}
dog(p)^{\mathsf{T}} & cat(p)^{\mathsf{T}} \\
\boxed{tall(p)} & \boxed{tall(p)} \\
tall(p)^{\mathsf{T}} & tall(p)^{\mathsf{T}} \\
\boxed{meow(p)} & \boxed{meow(p)} \\
meow(p)^{\mathsf{T}} & meow(p)^{\mathsf{T}} \\
(\forall x.dog(x) \Rightarrow \neg meow(x))^{\mathsf{T}} & \\
\neg meow(x)^{\mathsf{T}} & \\
meow(x)^{\mathsf{F}} & \\
\bot &
\end{array}
\end{array}$$

Grading:

AC1 only tableau
Points: 2
Feedback: The answer only contains a tableau no explanatory text of the steps

AC2 only extends branch 2
Points: 2.0
Feedback: The answer does not mention that branch 1 closes, and does not mention branch change, but extends branch 2.

AC3 only text
Points: 3.5

AC4 branch 2 steps unexplained
Points: -1
Feedback: The answer states that branch 1 closes and that new branch 2 is considered, but does not explain what happens there.

AC5 no branch change
Points: -2
Feedback: The answer states that the first branch closes, but does not mention what happens then.

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2. Give a representation of the model that the tableau machine predicts after seeing the discourse above and state where it comes from. 2 Points

Objective: apply model generation

Solution: We can read off the (Herbrand) model from the right branch, which is open:

$$\{cat(p), tall(p), meow(p)\}$$

Grading:

AC1 only tableau, no model
Points: 1

AC2 no model
Points: 0.0

AC3 partial first order model
Points: 1

AC4 peter is cat
Points: 1

4 Topics in Semantics

Problem 4.1 (Compositionality, Congruence, and Propositional Attitudes) 4 Points

1. Define the compositionality and congruence principles in general.

Objective: remember compositional

Objective: remember congruence principle

Solution: The compositionality principle states that the meaning of a complex expression is a function of the meanings of its sub-expressions, i.e. it only depends on those.
The congruence principle states that whenever A is part of B and A' means just the same as A, replacing A by A' in B will lead to a result that means just the same as B.

Grading:

AC1 no definition of compositionality
Points: -1

AC2 no definition of congruence
Points: -1

AC3 no operator meaning
Points: -0.5
Feedback: No mention of operator meaning that combines meaning of parts in the definition of compositionality.g

2. Discuss whether they hold when modeling propositional attitudes in natural language via modal logics; give counter-examples if one does not hold.

Objective: understand compositional
Objective: apply congruence principle

Solution: Modal logics are prominent meaning theories for propositional attitudes in natural language, most are compositional, but not do not obey congruence. The most prominent counter-example is that the ancient Greeks did not believe (a propositional attitude) that the morning star is the evening star even though the meaning of both is the planet Venus (as we now know).

Grading:

AC1 no counterexample.
Points: -1

AC2 no discussion
Points: -1

1. For compositionality, students of bring up the example that "every student sleeps" has the semantics of $\forall x.student(x) \Rightarrow sleep(x)$. Because this was introduced as an example "composition" in the introduction of LBS (along with abstraction and ambiguity)
2. For congruence, students often conflate equality of utterances and equality of meaning.

Problem 4.2
Consider the following two sentences:

E1 There is a dog that Peter does not like.

E2 Peter does not like all dogs.

1. Represent both of them in DRT.

4 Points
Objective: apply DRS

Solution:

1. $\begin{array}{|l|}\hline x \\ \hline \text{dog}(x) \\ (\neg\text{likes}(p, x)) \\ \hline \end{array}$

2. $\boxed{\begin{array}{l} \\ \hline (\neg\ \boxed{\begin{array}{l} \\ \hline (\boxed{\begin{array}{l} x \\ \hline \text{dog}(x) \end{array}} \Rightarrow \boxed{\begin{array}{l} \\ \hline \text{likes}(p, x) \end{array}}) \end{array}}\ ) \end{array}}$

Grading:

AC1 very wrong, but looks like DRSes
Points: 1

AC2 Formal errors
Points: -0.5
Feedback: Student's answer misses formal requirements.

AC3 DRS for E1 correct
Points: +1

AC4 DRS for E2 has no implication/negation structure
Points: -2

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2. Translate both into first-order logic using the standard DRT translation. 2 Points **Objective:** apply DRT translation

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Solution: The translations of the two DRSes above are logically equivalent:

1. $\exists x.\text{dog}(x) \wedge \neg\text{likes}(p, x)$
2. $\neg(\forall x.\text{dog}(x) \Rightarrow \text{likes}(p, x))$

Grading:

AC1 Very wrong, but looks like firstorder logic
Points: 0.5

AC2 no/wrong translation for E1
Points: -1

AC3 no/wrong translation for E2
Points: -1.5

AC4 no/wrong translation for E2
Points: -1

AC5 implication structure for E2, no wide negation
Points: -0.5

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3. Explain why the DRSes for the two sentences are should be different even though their translations to first-order logic are equivalent. 2 Points **Objective:** apply meaning theory **Objective:** apply semantic

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Solution: The example shows that the "representation level" (DRSes) of DRT adds value as a semantic meaning theory, as it can distinguish E1 and E2, which first-order logic cannot. E1 and E2 should be represented differently, as E1 can be continued with the sentence *It is a poodle.*, whereas E2 cannot.

Grading:

AC1 no mention of representational level and DRs
Points: -1

AC2 no discussion of why they should be different
Points: -1

Problem 4.3 (First-order Dynamic Logic DL^1)
Assume DL^1, extended with integers, equality, addition, and comparison operators with the expected semantics and consider the following DL^1 formulae:

$$\alpha = X{:=}2$$
$$\beta = X{:=}(X+1)\,;\,*X{:=}(X+1)$$
$$\gamma = (X<5)?$$

1. If we first execute α and then β, what are the possible values of X?

2 Points
Objective: apply first-order program logic

Solution: Any integer greater than 2.

Grading:

AC1 also executed gamma
Points: -0.5

AC2 included 2
Points: -0.5

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2. Find a value for φ such that the following formula is valid:

2 Points
Objective: apply first-order program logic

$$[\alpha\,;\,\beta\,;\,\gamma]\varphi \wedge \neg[\alpha\,;\,\beta]\varphi \wedge \neg[\alpha]\varphi$$

Solution: For example, φ could be $X=3 \vee X=4$.

Problem 4.4 (Kripke Semantics)

Objective: apply Kripke model

Consider

1. the set of possible worlds $W = \{v, w\}$
2. the propositional variables A and B
3. the variable assignment φ defined as follows:

$$\varphi(v,A) = \mathsf{T}$$
$$\varphi(v,B) = \mathsf{T}$$
$$\varphi(w,A) = \mathsf{T}$$
$$\varphi(w,B) = \mathsf{F}$$

1. State an accessibility relation $R \subseteq W \times W$, such that $\mathcal{I}_{\varphi}^{w}(\Box B) = \mathsf{T}$ 2 Points

   Objective: create accessibility relation

   Solution: E.g. $R = \{(v,v)\}$. In general, $(v,w) \notin R$ is necessary and sufficient.

2. Given a (new, arbitrary) accessibility relation R, provide an expression L such that $\mathcal{I}_{\varphi}^{v}(L) = \mathsf{T}$ if and only if $(v,w) \in R$. 2 Points

   Objective: apply modal interpretation

   Solution: For example, $L = \Diamond\neg B$ or $L = \neg(\Box B)$.