Assignment6 – Noun Phrases in Fragment 4

Problem 6.1

Consider a Kripke model, where the worlds are the natural numbers (including 0) and a is accessible from b if $a \geq b$. Furthermore, let A, B, and C be propositional variables, where

1. A holds in a world w iff w is even,
2. B holds in w iff w is odd,
3. C only holds in world 5, and
4. D only holds in world 8.

Which of the following formulas are valid in this Kripke model?

1. $C \Rightarrow A$
2. $C \Rightarrow B$
3. $C \Rightarrow D$
4. $C \Rightarrow \Box A$
5. $C \Rightarrow \Box B$
6. $C \Rightarrow \Box D$
7. $C \Rightarrow \Diamond A$
8. $C \Rightarrow \Diamond B$
9. $C \Rightarrow \Diamond D$
10. $C \Rightarrow \Diamond\Diamond D$
11. $C \Rightarrow \Diamond\Box D$
12. $C \Rightarrow \Box\Diamond D$
13. $C \Rightarrow \Box\Box D$
14. $\Diamond A$
15. $\Box A$
16. $\Diamond C$
17. $\Box C$
18. $\Diamond C \Rightarrow \Diamond D$

19. $\Diamond D \Rightarrow \Diamond C$
20. $\Box C \Rightarrow \Diamond D$
21. $\Diamond C \Rightarrow \Box D$

Problem 6.2

Objective: understand *Kripke model*

In *(propositional) modal logic*, there is a correspondence between *Kripke models* and the axioms of the logic. For example, if the accessibility relation is reflexive and transitive, then the the following axiom is valid (among others):

$$\Box A \Rightarrow \Box\Box A$$

1. Demonstrate that the axiom is not valid if the accessibility relation is not transitive.

Hint: Provide a *Kripke model* as a counterexample.

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2. Demonstrate that the axiom must be valid for any Kripke model with a reflexive and transitive accessibility relation.
3. Is reflexivity necessary for the axiom to be valid or is transitivity alone sufficient?