

Assignment5 – Lambdas and Fragment 5

Problem 5.1 (Church Booleans)

Boolean logic can be encoded in the lambda calculus. Concretely, we will denote truth values as functions:

$$false := \lambda x.\lambda y.y$$

$$true := \lambda x.\lambda y.x$$

1. Let

$$or := \lambda a.\lambda b.a a b$$

Compute $or\ false\ true$, i.e. beta reduce

$$(\lambda a.\lambda b.a a b) (\lambda x.\lambda y.y) (\lambda x.\lambda y.x)$$

Hint: The result should be true, i.e. a function of the form $\lambda x.\lambda y.x$.

2. What should be the function for the *not* operator?

Hint: It helps to have a feeling for *true* and *false*. They are both functions that accept two arguments, x and y . *true* returns the first argument, x , while *false* returns the second argument, y .

The function *or* maps two booleans, a and b to the expression $a a b$. The “top level” function is a , and it gets the arguments a and b . If a is true, the expression will evaluate to the first argument, a . In other words, if a is true, the expression will evaluate to true. If a is false, the expression will evaluate to the second argument, b . So if both a and b are false, it will evaluate to false, but if b is true, it will evaluate to true.

We could also have defined *or* as

$$or = \lambda a.\lambda b.a\ true\ b$$

3. What should be the function for the *nor* operator ($X\ nor\ Y$ is true iff neither X nor Y are true)?

Problem 5.2 (Church Booleans)

Given the following lambda expressions

$$\begin{aligned} & p (f x x) \\ & p (h (p y)) \\ & p ((\lambda k.g k)x) \\ & p (m (\lambda l.p l)) \end{aligned}$$

and the types

$$\begin{aligned} p & : \iota \rightarrow o \\ x & : \iota \end{aligned}$$

What types must the other symbols have for the expressions to type-check?

Problem 5.3

Consider the following *sentence*:

Ethel didn't scream.

1. How would the *sentence* be *parsed* according to the grammar of *Fragment 3*?
2. Apply the *translation rules/semantics construction* from *Fragment 3* to the *sentence*.

Objective: apply *grammar*

Objective: apply *translation rule*

Hint: The translation rules are not clearly specified in the slides. Try to understand the ideas behind fragment 3 to fill in the gaps.
