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General Computer Science I (320101) Fall 2012
Quiz 10: Hilbert Calculus
(Given Nov. 26. 2012)

12pt

Problem 10.1 (Hilbert Calculus)

Consider a calculus given by the axioms

1. **K** := $P \Rightarrow Q \Rightarrow P$,
2. **S** := $(P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R$

and the following rules:

$$\frac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{A}}{\mathbf{B}} \text{MP} \qquad \frac{\mathbf{A}}{[\mathbf{B}/\mathbf{X}](\mathbf{A})} \text{Subst}$$

Prove that $(M \Rightarrow N) \Rightarrow M \Rightarrow M$

Solution:

1. Applying the substitutions $[M/P], [N/Q], [M/R]$ to axiom **S** yields

$$(M \Rightarrow N \Rightarrow M) \Rightarrow (M \Rightarrow N) \Rightarrow M \Rightarrow M$$

2. Applying the substitutions $[M/P], [N/Q]$ to axiom **K** yields $M \Rightarrow N \Rightarrow M$
 3. Applying MP on 1. and 2. gives $(M \Rightarrow N) \Rightarrow M \Rightarrow M$
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