General Computer Science I (320101) Fall 2012
Tutorial 1: Math Tutorial

2 The role of formal logic in mathematics

If looking at a mathematical text, we will find that terms and statements are deduce from other terms and statements. To emphasize this process, mathematicians use word such as "therefore, consequently, due to, because of, accordingly, ...". The terms and statements are not incoherently placed next to each other, but are rather arranged and linked by logical conclusions. Statements are marked as consequence of other statements. The insight in their relationship is conveyed by the proof. The logic is the frame, which is needed to construct the whole (a theory).

Statements A statement shall be defined as something that is either true or false, and not both at the same time, but at least either one. Statement are:

- Berlin is in Europe
- Snow is black
- $3 + 2 = 6$
- $2H_2 + 0_2 \rightarrow 2H_2O$

65pt

Problem 2.1 (Mathematical Statements)
Which of the following sentences are mathematical statements?Which statements are true? Which additional assumptions are needed for some of the statements, to decide whether they are true or false?

1. The moon is a green cheese.
2. Karl the Great used an electric razor.
3. All humans that are 2m tall are older then 200 years.
4. This statement is false.
5. This sentences is no statement.
6. The following sentences is false. The previous sentences is true.
7. Every sentences in this exercise is false.
8. He likes to read books.
9. All Knaffs contain green Hunkis.

10. The 54th letter of this exercise is an “e”
11. The human is a unfeathered biped.
12. $2^{2^{17}}$ is a prime number

Solution:

- wrong statements: 1,2,3
- true statements: 11
- statements with unknown logical value: 10, 12
- no statement: 4,5,6,7 (are true and false at the same time, lead to contradiction)
- 9 is grammatical correct, but without any sense
- 8 is a function

Junction: Conjunction, Disjunction, Negation, Implication, Equivalence TODO: give an introduction in junctions with examples. 65pt

Problem 2.2 (Junctions)

The junction $A|B$ is described by the following truth table:

A	B	$A\|B$
w	w	f
w	f	w
f	w	w
f	f	w

What kind of junction you know are the following two examples: $A|(A|B)$ and $(A|B)|(A|B)$

Solution:

- $A|B$ equals $(\neg A) \vee (\neg B) \rightarrow$ (not A) or (not B)
- $A|(A|B)$ is $A \rightarrow B$
  in detail: $A|(A|B) = \neg A \vee \neg(A|B) = \neg A \vee \neg(\neg A \vee \neg B) = \neg A \vee A \vee B = A \rightarrow B$
- $(A|B)|(A|B)$ is $A \wedge b$
  in detail: $(A|B)|(A|B) = \neg(\neg A \vee \neg B) \vee \neg(\neg A \vee \neg B) = A \wedge B \vee A \wedge B = A \wedge B$

Problem 2.3 (Junctions) 65pt

Which of the following junction is true?

1. If I am not older then 200 yeas, then I am taller then 2 m.
2. If a human is taller then 2m, then he is also older than 200 years.
3. Only if a human is taller then 2m, then he is older than 200 years.
4. Either $5 < 3$ or from $2 + 3 = 5$ implies $3 * 4 = 12$
5. Either $5 < 3$ or from $2 + 3 = 6$ implies $3 * 4 = 12$
6. Either $5 > 3$ or from $2 + 3 = 6$ implies $3 * 4 = 12$
7. If $5 > 3$, then $2 + 3 = 6$ as well as $3 * 4 = 12$
8. If $5 > 3$, then $2 + 3 = 5$ as well as $3 * 4 = 12$
9. I am tall and I am small.
10. I am tall or I am small.
11. 7 or 9 are prime numbers.
12. 2 and 4 are factors of 216.
13. 2 or 4 are factors of 216.
14. either 2 or 4 are factors of 216.

Solution:

1. false, since $\neg A \vee B$: "I am older then 200 years or I am taller then 2 m"
2. true, "smaller than 2m or older then 200 years"
3. true
4. true
5. true
6. false
7. false
8. true
9. true
10. false

11. true
12. true
13. true
14. true
15. false
