

Midterm Grand Tutorial

General CS I (320101)

October 25, 2014

1 Mathematical Foundations

Problem 1.1 (Nicely Greater Sets)

Consider two arbitrary finite sets A and B of real numbers. We say that A is *nicely greater* than B , denoted $A \gg B$, iff the following conditions are fulfilled:

1. $\#(A) > \#(B)$
2. $\forall a \in A \setminus B. \forall b \in B. a > b$

Prove or refute that the relation \gg is transitive.

Example: For $A = \{1, 2, 3\}$, $B = \{1, 2\}$ and $C = \{2, 4\}$, we only have $A \gg B$.

Note: $\#(A)$ is the size of the set A .

Problem 1.2 (UNN practice)

Let's redefine the successor function for UNN as follows:

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$$s(0) = 1$$

$$s(n) = n^2 - 3n + 2, n > 0$$

Using this definition, write down the set of unary natural numbers.

Do the first four Peano axioms still hold? If not, which one fails and why?

2 Sets, Relations and Functions

Problem 2.1 (Injective functions)

Let f be a function $f: U \rightarrow V$, where U and V are not-empty sets. Prove or refute that f injective iff there is a function $g: V \rightarrow U$ with $g \circ f = \text{Id}_U$.

5min

Problem 2.2: Prove or refute the following statements. If you want to refute it, a counterexample is enough.

1. Define a character code from $\mathbb{D} = \{0, 1 \dots 9\}$ to $\mathbb{B} = \{0, 1\}$
 $c : \mathbb{D} \rightarrow \mathbb{B}^+$; $(n)_{10} \mapsto (n)_2$. ie. $c(1) = 1, \dots c(7) = 111, c(8) = 1000, c(9) = 1001$
 The string code generated by this character code c is injective.
2. $f : \mathbb{R} \rightarrow \mathbb{R}; n \mapsto 2^n$ is surjective.

Problem 2.3 (Functions practice)

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1. Prove or refute

The function $f : \mathbb{R} \rightarrow \mathbb{R}, f(x) = \frac{e^x}{\sqrt{x^3 + 4}}$ is a **total function**.

2. Prove or refute bis

The function $g : \mathbb{R} \rightarrow \mathbb{R}, g(z) = \sqrt{z}$ is a **partial function**.

3 Abstract Data Types

Problem 3.1 (Base 10 Natural Numbers)

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A natural number in base 10 is any combination of digits between 0 and 9. Construct an ADT that represents natural numbers in base 10. Please note that leading zeros are **not** allowed. Represent the number 472 with your ADT.

Hint: Treat 0 as a separate sort.

4 Programming in Standard ML

Problem 4.1 (Crates at sea)

A certain ship is at sea on a voyage from Bremen to New York. There is a crate on her deck. However the crate has not been attached/fixed properly and when in a slight storm the ship goes over a wave, the crate can move one meter to the right or to the left. When it reaches the edge of the deck it is stopped by the rails. Assume that the crate is originally in the middle of the deck and that it is 1 meter wide. Write an SML function that given the half the decks width (so the distance from the original position of the crate to the edge of the deck) and a string describing the crates movements, determines how many times the crate will reach the edges of the deck. The string consists of the words "right", "left" for the respective movements. (You can ignore any other characters in between the words).

The function will have the following signature:

val count = **fn** : string * int -> int

Example:

```
– count("right left left left left_left_a_ right right right right –a–  
right_ right right right right left right move left",4);  
val it = 3 : int
```

Problem 4.2 (Angry Birds)

Your new favorite game is Angry Birds, and, after a lazy afternoon when you have played 0pt the game, you observed the following rules for deducing the score:

- the red bird will always add 5000 points to your score (no matter what it hits)
- the blue bird always is split into 3 smaller birds, and every bird adds 1000 points if it hits an object
- the yellow bird will add 2500 birds only if it hits a green pig
- the white bird is considered peaceful and will add no points to your score

Your version of Angry Birds permits you to choose K from a list of birds which you will fire. Therefore, you design the following SML datatype:

```
datatype angrybird = white | red | blue of int | yellow of bool;
```

The int parameter of the blue bird tells you how many birds hit an object, and the bool parameter of the yellow bird tells you if the bird hits a pig or not.

You are required to write an SML function `getScore` which takes a list of Angry Birds and the number K of birds you can fire and returns the maximum score you can get by firing your choice of K birds:

```
val getScore = fn : angrybird list * int -> int;  
– getScore( [white, white, red, blue(3)], 3 );  
val it = 8000 : int; (* red, blue and one of the whites are chosen *)  
– getScore( [red, yellow(false), yellow(true), blue(2)], 2);  
val it = 7500 : int; (* red and yellow(true) are chosen *)
```

Hint: For sorting a list of ints in descending order, you can use the following SML snippet:

```
– ListMergeSort.sort (op<) [1,5,1,2];  
val it = [5,2,1,1] : int list;
```

5 Formal Languages

Problem 5.1: Given the alphabet $A = \{0, 1\}$ and a $L = \cup_{i=1}^{\infty} L_i$ where 0pt

- $L_1 = \{\epsilon\}$
- $L_n = \{0x1, 1x0, 0x0, 1x1 \mid x \in L_{n-1}\}$

1. Write out explicitly what is in L_3 . Show L contains all the words of even length.
2. Give another inductive definition of the language which contains all the words of even length. (Other than what is shown in the question, of course.)