General Computer Science (320101) Fall 2015

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For Course Purposes Only

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Contents

Assignment 1 (Elementary Math) Given Sep. 11., Due Sep. 18.

Problem 1.1 (A wrong induction proof)
What is wrong with the following "proof by induction"? 20pt

> **Theorem:** All students of Jacobs University have the same hair color.
>
> **Proof:** We prove the assertion by induction over the number n of students at Jacobs University.
>
> **base case:** $n = 1$. If there is only one student at Jacobs University, then the assertion is obviously true.
>
> **step case:** $n > 1$. We assume that the assertion is true for all sets of n students and show that it holds for sets of $n + 1$ students. So let us take a set S of $n + 1$ students. As $n > 1$, we can choose students $s \in S$ and $t \in S$ with $s \neq t$ and consider sets $S_s = S\backslash\{s\}$ and $S_t := S\backslash\{t\}$. Clearly, $\#(S_s) = \#(S_t) = n$, so all students in S_s and have the same hair-color by inductive hypothesis, and the same holds for S_t. But $S = S_s \cup S_t$, so any $u \in S$ has the same hair color as the students in $S_s \cap S_t$, which have the same hair color as s and t, and thus all students in S have the same hair color □

Problem 1.2 (Drive safely)
n cars are travelling down a narrow one-way street. We know that: 20pt

- The distance d between each two cars is the same.
- The safe breaking distance b is the minimum distance between two cars that is needed for the second car to stop on time if the car in front suddenly breaks.
- $d < b$

Prove by induction or refute: if the first car suddenly stops moving, all cars will stop moving. Before you do the induction state the property P you are using in the induction axiom.