Quizzes for General CS (CH08-320101) Fall 2016

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For Course Purposes Only

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Contents

1 Quiz 1 (Introductory Quiz) – Given Sep. 6. 2016

Problem 1.1 (GenCS Grading)
State the components of the overall grade of the GenCS course and discuss their intention. 6pt

2 Quiz 2 (Peano Axioms Quiz) – Given Sep. 13. 2016

Problem 2.1 (Natural numbers)
Prove or refute that $s(s(o))$ and $s(s(s(o)))$ are unary natural numbers and that their successors are different. 6pt

Problem 2.2 (Peano's induction axiom)
State Peano's induction axiom and discuss what it can be used for. 6pt

3 Quiz 3 (Sets and Mathtalk Quiz) – Given Sep. 20. 2016

Problem 3.1 (Talking about Sets)

8pt

Given the following sets

1. $A = \{a, b, c, d, e\}$
2. $B = \{d, f, h\}$
3. $C = \{d, f, g, i\}$

Define each of the following operations on sets **in math talk** and apply it to the given sets:

1. intersection (general formula in math talk):
   $A \cap B =$
2. union (general formula in math talk):
   $B \cup C =$
3. set difference (general formula in math talk):
   $A \backslash B =$
4. Cartesian product (general formula in math talk):
   $B \times C =$

Problem 3.2 (Greek alphabet)

4pt

Fill in the blanks in the following table of Greek letters. Note that capitalized names denote capital Greek letters.

Symbol		Φ		Γ		Σ		θ		χ
Name	zeta		pi		Psi		eta		Omega	

4 Quiz 4 (Relations) – Given Sep. 27. 2016

Problem 4.1 Given set $A = \{a, b, c, d, e, f, g\}$ determine whether the following relations $R \subseteq A \times A$, are reflexive, symmetric and/or transitive and briefly justify your answer: 8pt

- $R = \{\langle a, a\rangle, \langle c, d\rangle, \langle a, c\rangle, \langle b, d\rangle\}$
- $R = \{\langle a, a\rangle, \langle g, g\rangle, \langle e, e\rangle, \langle d, d\rangle\ \langle a, c\rangle\ \langle c, c\rangle\ \langle c, a\rangle\ \langle c, d\rangle\ \langle f, f\rangle\ \langle b, b\rangle\}$
- $R = \{\langle b, e\rangle\ \langle e, b\rangle\ \langle d, d\rangle\ \langle d, f\rangle\ \langle c, g\rangle\ \langle f, d\rangle\ \langle g, c\rangle\ \}$

Give an example of a relation on the given set A which is a strict partial order and which has at least three elements.

Problem 4.2 (Function Definition)
Let A and B be sets. State the definition of the concept of a partial function with domain A and codomain B. Also state the definition of a total function with domain A and codomain B. 4pt

5 Quiz 5 (Functions) – Given Oct. 4. 2016

Problem 5.1 Given the set $A = \{1, 2, 3, 4\}$ 9pt

1. Write down a total function $f\colon A \to A$ which is also an antisymmetric but not reflexive relation.
2. Give the converse relation of your function f and prove whether it is a total function, partial function, or not a function at all.

Problem 5.2 Let $\mathbb{A} := \{a, b\}$ and $\mathbb{C} := \{c, d\}$, write down the function space $\{f \mid f\colon \mathbb{C} \to \mathbb{A}\}$. 3pt

6 Quiz 4 (SML Language) – Given Oct. 11. 2016

Problem 4.1 6pt

```
val merge = fn : 'a list * 'a list -> 'a list
- merge([1,3,5],[2,4,6]);
val it = [1,2,3,4,5,6] : int list
```

Write an SML function *merge* that takes two **sorted** lists and merges them into another **sorted** list.

Hint: Recall that we call a list $[l_1,l_2,\ldots,l_n]$ **sorted**, iff it is empty, a singleton list, or $l_i \leq l_{i+1}$ for all $1 \leq i \leq n-1$.

Problem 4.2 (Square the list)

Write an SML function squareList that takes an int list and returns the list with every element squared. You are required to use a higher-order function, solutions without use of higher-order function will not be given full points. 6pt

Example:

```
- squareList [1, 4, 3, 10];
val it = [1, 16, 9, 100] : int list;
```

Note: $x^2 = x * x$

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7 Quiz 7 (Abstract Data Types) – Given Oct. 18. 2016

Problem 7.1 A binary tree is a tree in which each node has either 2 or no children. A node that has no children is called a leaf. Construct an abstract data type for binary trees. Each node needs to store a natural number as well. Give a ground constructor term that represents a binary tree with at least four nodes for your ADT. 12pt

8 Quiz 8 (Mutual Recursion) – Given Nov. 1. 2016

Problem 8.1 (Programming with effects)

12pt

1. Write down two SML functions that compute the following sequences. Make sure that the user does not try to access a negative index (i.e. $n < 0$) or the square root takes a negative argument by raising exceptions.

$$a_n := \begin{cases} 2 & \text{if } n = 0 \\ \sqrt{b_{n-1}} + a_{n-1} & \text{else} \end{cases} \qquad b_n := \begin{cases} 1 & \text{if } n = 0 \\ 3a_{n-1} - b_{n-1} & \text{else} \end{cases}$$

Note: The two sequences are mutually recursive, i.e. they call each other.

Hint: You can use the library function Math.sqrt for computing the square root.

2. Explain what would happen if you would not use exceptions in your functions from the point of view of termination.

9 Quiz 9 (Formal Languages) – Given Nov. 8. 2016

Problem 9.1 (Formal language)
Give the definition of the formal language of the words over $\{0,1\}$ that are palindromes. 6pt

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Note: A palindrome is a word w that is identical to w reversed. For example: "1001" and "0010100".

Problem 9.2 Let $A := \{a,b,c,d,e,f,g,h\}$ and $\mathbb{B} := \{0,1\}$, and 6pt

$c(a)$:=010010010101001	$c(b)$:=010110010101001
$c(c)$:=010011110101001	$c(d)$:=010010011101001
$c(e)$:=010010010110001	$c(f)$:=010010010101101
$c(g)$:=010011110101000	$c(h)$:=011111110101000

Is c a character code? Does it induce a code on strings? Justify your answers to both questions.

10 Quiz 10 (Boolean Expressions) – Given Nov. 15. 2016

Problem 10.1 (Evaluating Expressions)

Given the expression $E := \overline{x_0 * \overline{x_1}} + x_2$, your tasks are: 12pt

1. If $\varphi := [\mathsf{F}/x_0], [\mathsf{T}/x_1], [\mathsf{F}/x_2]$, evaluate the expression using the evaluation function $\mathcal{I}_\varphi(E)$ and showing the whole computation.
2. Write down the truth table for the expression.
3. Specify cost and depth of the expression.

11 Quiz 9 (Quine-McCluskey Algorithm) – Given Nov. 22. 2016

Problem 9.1 (Quine-McCluskey)
Use the algorithm of Quine-McCluskey to determine the minimal polynomial of the following function: 12pt

$x1$	$x2$	$x3$	f
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

12 Quiz 11 (Propositional Logic) – Given Nov. 29. 2016

Problem 11.1 (Hilbert calculus)
Consider the Hilbert-style calculus given by the axioms 9pt

$$K := P \Rightarrow Q \Rightarrow P \qquad S := (P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R$$

and the rules:

$$\frac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{A}}{\mathbf{B}}\,\text{MP} \qquad \frac{\mathbf{A}}{[\mathbf{B}/X](\mathbf{A})}\,\text{Subst}$$

Prove that $(\mathbf{M} \Rightarrow \mathbf{M}) \Rightarrow \mathbf{M} \Rightarrow \mathbf{M}$.

Problem 11.2 (An incorrect calculus)
Why is this calculus $\mathcal{C}^2$ incorrect? 3pt

- $\mathcal{C}^2$ Axiom: $P \Rightarrow Q \Rightarrow P$
- $\mathcal{C}^2$ Inference Rules: $\dfrac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{B} \Rightarrow \mathbf{C}}{\mathbf{C}}\,R2$ $\qquad \dfrac{\mathbf{A}}{[\mathbf{B}/P]\mathbf{A}}\,\text{Subst}$