Quizzes for General CS (320101) Fall 2015

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For Course Purposes Only

December 12, 2016

Contents

1 Assignment 1 (Introductory Quiz) – Given Sep. 8. 2015

Problem 1.1 (What is an algorithm?)
What is an algorithm? Give 3 examples of algorithms and explain them (be creative and make sure that at least two of them are not on the slides!). 9pt

Problem 1.2 (GenCS Grading)
State the components of the overall grade of the GenCS course and discuss their intention. 3pt

2 Assignment 1 (Unary Natural Numbers & Induction) – Given Sep. 14. 2015

Problem 1.1 (Three is a unary natural number)

Using only the Peano axioms as discussed in class, prove that $s(s(s(o)))$ is a unary natural number. 4pt

Problem 1.2 (Peano's induction axiom)

State Peano's induction axiom and discuss what it can be used for. 8pt

3 Assignment 3 (Relations) – Given Sep. 22. 2015

Problem 3.1 (Greek Letters)
Fill in the blanks in the following table of Greek letters. Note that capitalized names denote capital Greek letters. 2pt

Symbol		Ξ	η			Λ		ω	I	
Name	delta			sigma	Psi		Omega			chi

Problem 3.2 (Talking about Sets)

4pt

Given the following sets

1. $A = \{a, b, c, d, e\}$
2. $B = \{d, f, h\}$
3. $C = \{d, f, g, i\}$

Define each of the following operations on sets **in math talk** and apply it to the given sets:

1. intersection:$S \cap T :=$
   e.g. $A \cap B =$
2. union: $S \cup T :=$ e.g. $B \cup C =$
3. set difference: $S \backslash T :=$ e.g. $A \backslash B =$
4. n-fold Cartesian product: $S_1 \times \ldots \times S_n :=$ e.g. the size $\#(A \times B \times C) =$

Problem 3.3 (Relation Properties)
You are given the set $S := \{a, b, c, d\}$ and the relation 6pt

$$R \subseteq S \times S, \quad R := \{(a, a), (a, c), (b, b), (b, d), (c, a), (c, c), (c, d), (d, b), (d, c), (d, d)\}$$

Determine whether R is *a)* reflexive, *b)* symmetric, *c)* transitive, or *d)* antisymmetric. If the relation does not have a certain property, give a counter-example to show that.

4 Assignment 4 (Functions) – Given Sep. 29. 2015

Problem 4.1 (Function properties)

10pt

1. State in mathtalk the definition of an injective total function (you may take the concept of a function as given).
2. Determine whether the function $f\colon \mathbb{N} \to \mathbb{N}$ with $f(n) := 2n$ is injective, surjective or bijective.

Problem 4.2 (Lambda Notation)

Write down the function $f\colon \mathbb{R} \to \mathbb{R}$ with $f(x) = \sqrt{x}$ using lambda-notation. 2pt

5 Assignment 5 (SML Basics) – Given Oct. 6. 2015

Problem 5.1 (Arithmetic Mean)

Write an SML function `am = fn : real list -> real` that given a list of real numbers returns the arithmetic mean. 12pt

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Hint: The arithmetic mean of a sequence $a_1, a_2, \ldots a_n$ is defined as $\frac{1}{n}\sum_{i=1}^{n} a_i$. For $n = 0$ your function should return `0.0`.

Example: `am [1.0, 3.0, 3.0, 2.0]` should return `2.25`.

6 Assignment 6 (SML Datatypes) – Given Oct. 13. 2015

Problem 6.1 (Temperatures)

You are given the following SML datatype temp that represents temperatures in Fahrenheit and Celsius. 12pt

```
datatype temp = Celsius of real | Fahrenheit of real;
```

Write an SML function find : temp list -> temp that returns the lowest temperature in a list. For instance,

```
- find([Celsius(12.0), Fahrenheit(52.0), Celsius(32.0)]);
val it = Fahrenheit 52.0 : temp
```

Note: You can use the following formula for transforming Fahrenheit into Celsius: $t_C = (t_F - 32) \cdot \frac{5}{9}$

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7 Assignment 7 (Formal Languages) – Given Nov. 3. 2015

Problem 7.1 (Formal Languages and Lexical Order)
Given the alphabet $A = \{a, b, c\}$ and 2 formal languages in A 12pt
a. $L_1 = \{a^{[n]}b \mid n \in \mathbb{N}\}$ and
b. $L_2 = \{c^{[n]}bc^{[n]} \mid n \in \mathbb{N}\}$
write down 3 different strings $s_1, s_2, s_3 \in \text{conc}(L_1, L_2)$. What is the lexical order of s_1, s_2, s_3 given that $a \prec^A b \prec^A c$?

8 Assignment 8 (Formal Languages) – Given Nov. 10. 2015

Problem 8.1 (Formal language)
Give the definition of the formal language of the words over $\{0, 1\}$ that are palindromes. 4pt

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Note: A palindrome is a word w that is identical to w reversed. For example: "1001" and "0010100".

Problem 8.2 Given the alphabet $A = \{a, b, c\}$ and a $L := \bigcup_{i=1}^{\infty} L_i$, where $L_1 = \{\epsilon\}$ and L_{i+1} contains the strings x, bbx, xac for all $x \in L_i$. 8pt

1. Is L a formal language?
2. Which of the following strings are in L? Justify your answer

$s_1 = bbac$	$s_2 = bbacc$	$s_3 = bbbac$
$s_4 = acac$	$s_5 = bbbacac$	$s_6 = bbacac$

9 Assignment 9 (Boolean Logic) – Given Nov. 17. 2015

Problem 9.1 (CNF and DNF)

12pt

1. Find the CNF and DNF of the boolean function that corresponds to the expression

$$\overline{\overline{x_1} * x_2 + x_3}$$

2. Identify the cost and depth of this expression.

10 Assignment 10 (QMC) – Given Nov. 24. 2015

Problem 10.1 (Quine-McCluskey)
Use the algorithm of Quine-McCluskey to determine the minimal polynomial of the following function: 6pt

$x1$	$x2$	$x3$	f
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

Problem 10.2 For each of the following propositional formulae, state (and justify!) whether they are satisfiable/falsifiable/unsatisfiable/valid. 6pt

1. $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow p \Rightarrow r$
2. $x \wedge \neg (x \vee y)$
3. $\text{love}(\text{bill}, \text{mary}) \wedge \text{love}(\text{mary}, \text{bill}) \Rightarrow \text{love}(\text{bill}, \text{bill})$

Note: You can use whichever method you like: truth tables, boolean algebra, or anything else we introduced in the lecture.

11 Assignment 11 (Propositional Logic) – Given Dec. 1. 2015

Problem 11.1 (Hilbert calculus)
Consider the Hilbert-style calculus given by the axioms 9pt

$$K := P \Rightarrow Q \Rightarrow P \qquad S := (P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R$$

and the rules:

$$\frac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{A}}{\mathbf{B}}\,\text{MP} \qquad \frac{\mathbf{A}}{[\mathbf{B}/X](\mathbf{A})}\,\text{Subst}$$

Prove that $(\mathbf{M} \Rightarrow \mathbf{M}) \Rightarrow \mathbf{M} \Rightarrow \mathbf{M}$.

Problem 11.2 (An incorrect calculus)
Why is this calculus $\mathcal{C}^2$ incorrect? 3pt

- $\mathcal{C}^2$ Axiom: $P \Rightarrow Q \Rightarrow P$
- $\mathcal{C}^2$ Inference Rules: $\dfrac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{B} \Rightarrow \mathbf{C}}{\mathbf{C}}\,R2$ $\qquad \dfrac{\mathbf{A}}{[\mathbf{B}/P]\mathbf{A}}\,\text{Subst}$