Quizzes for General CS II (320102) Fall 2014

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Contents

1 Assignment 1 (Introductory Quiz) – Given Sep. 8. 2014

Problem 1.1 (Keywords of General Computer Science)
Our course started with a motivation of "General Computer Science" where some fundamental notions where introduced. Name three of these fundamental notions and give for each of them a short explanation. 8pt

Problem 1.2 (GenCS Grading)
State the components of the overall grade of the GenCS course and discuss their intention. 4pt

2 Assignment 2 (Unary Natural Numbers) – Given Sep. 15. 2014

Problem 2.1 (UNN Powers)
Give the defining equations for the the power operation $\pi\colon \mathbb{N}_1 \times \mathbb{N}_1 \to \mathbb{N}_1$ on unary natural numbers. Assume the addition $\alpha\colon \mathbb{N}_1 \times \mathbb{N}_1 \to \mathbb{N}_1$ and multiplication $\mu\colon \mathbb{N}_1 \times \mathbb{N}_1 \to \mathbb{N}_1$ operations are already given. 8pt

Problem 2.2 (Peano's induction axiom)
State Peano's induction axiom and discuss what it can be used for. 4pt

3 Assignment 3 (Relations and functions) – Given Sep. 22. 2014

Problem 3.1 Given set $A = \{a, b, c, d, e, f, g\}$ determine whether the following relations $R \subseteq A \times A$, are reflexive, symmetric and/or transitive and briefly justify your answer: 6pt

- $R = \{\langle a, a\rangle, \langle c, d\rangle, \langle a, c\rangle, \langle b, d\rangle\}$
- $R = \{\langle a, a\rangle, \langle g, g\rangle, \langle e, e\rangle, \langle d, d\rangle \langle a, c\rangle \langle c, c\rangle \langle c, a\rangle \langle c, d\rangle \langle f, f\rangle \langle b, b\rangle\}$
- $R = \{\langle b, e\rangle \langle e, b\rangle \langle d, d\rangle \langle d, f\rangle \langle c, g\rangle \langle f, d\rangle \langle g, c\rangle\ \}$

Give an example of a relation on the given set A which is a strict partial order and which has at least three elements.

Problem 3.2 (Function Definition)
Let A and B be sets. State the definition of the concept of a partial function with domain A and codomain B. Also state the definition of a total function with domain A and codomain B. 6pt

4 Assignment 4 (SML Language) – Given Sep. 29. 2014

Problem 4.1 8pt

```
val merge = fn : 'a list * 'a list -> 'a list
- merge([1,3,5],[2,4,6]);
val it = [1,2,3,4,5,6] : int list
```

Write an SML function *merge* that takes two **sorted** lists and merges them into another **sorted** list.

Hint: Recall that we call a list $[l_1,l_2,\ldots,l_n]$ **sorted**, iff it is empty, a singleton list, or $l_i \leq l_{i+1}$ for all $1 \leq i \leq n-1$.

Problem 4.2 (Add elements of list)

Implement a function that given an int list outputs the sum of its elements with the following signature and example: 4pt

```
val sum = fn : int list -> int
- sum[0,3,2,5];
val it = 10 : int
```

5 Assignment 6 (Abstract Data Types) – Given Oct. 13. 2014

Problem 6.1 A binary tree is a tree in which each node has either 2 or no children. A node that has no children is called a leaf. Construct an abstract data type for binary trees. Each node needs to store a natural number as well. Give a ground constructor term that represents a binary tree with at least four nodes for your ADT. 12pt

6 Assignment 7 (Character Codes) – Given Nov. 3. 2014

Problem 7.1 (Character Encodings)

12pt

Briefly introduce and discuss the relative merits of

1. the `ASCII` code,
2. the ISO-Latin codes,
3. the Universal Character Set, and
4. the Unicode encodings UTF-8, UTF-16, and UTF-32

7 Assignment 8 (Boolean expressions) – Given Nov. 10. 2014

Problem 8.1 (Evaluating Expressions)

Given the expression $E := (x_0 + x_1) * (\overline{x_1} + x_0 * x_2)$ 12pt

Your tasks are:

1. If $\varphi := [\mathsf{F}/x_0], [\mathsf{T}/x_1], [\mathsf{F}/x_2]$, evaluate the expression using the evaluation function $\mathcal{I}_\varphi(E)$ and showing the whole computation.
2. Write down the truth table for the expression.
3. What is the depth of the expression?

8 Assignment 9 (Quine-McCluskey Algorithm) – Given Nov. 17. 2014

Problem 9.1 (Quine-McCluskey)
Use the algorithm of Quine-McCluskey to determine the minimal polynomial of the following function: 12pt

$x1$	$x2$	$x3$	f
F	F	F	F
F	F	T	T
F	T	F	F
F	T	T	T
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	F

9 Assignment 10 (Hilbert Calculus) – Given Nov. 24. 2014

Problem 10.1 (A Hilbert Calculus)

Consider the Hilbert-style calculus given by the axioms 12pt

1. $K := P \Rightarrow Q \Rightarrow P$
2. $S := (P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R$

and the rules:

1. $\dfrac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{A}}{\mathbf{B}}$ MP
2. $\dfrac{\mathbf{A}}{[\mathbf{B}/X](\mathbf{A})}$ Subst

Prove that $((\mathbf{A} \Rightarrow \mathbf{B} \Rightarrow \mathbf{C}) \Rightarrow (\mathbf{A} \Rightarrow \mathbf{B})) \Rightarrow (\mathbf{A} \Rightarrow \mathbf{B} \Rightarrow \mathbf{C}) \Rightarrow \mathbf{A} \Rightarrow \mathbf{C}$.

Hint: Look at the given rules and find out which one is better suited for starting the proof.

10 Assignment 11 (Hilbert Calculus) – Given Dec. 1. 2014

Problem 11.1 (Natural Deduction)

12pt

Given the following inference rules for $\mathcal{ND}^0$:

Introduction	Elimination	Elimination	Implication
$\dfrac{\mathbf{A} \quad \mathbf{B}}{\mathbf{A} \wedge \mathbf{B}} \wedge I$	$\dfrac{\mathbf{A} \wedge \mathbf{B}}{\mathbf{A}} \wedge E_l$	$\dfrac{\mathbf{A} \wedge \mathbf{B}}{\mathbf{B}} \wedge E_r$	$\dfrac{\overline{[\mathbf{A}]^1}}{\mathbf{B}} \Rightarrow I^1$

Prove that $\mathbf{A} \wedge (\mathbf{B} \wedge \mathbf{C}) \Rightarrow \mathbf{C} \wedge \mathbf{A}$. Specify the rules applied at each step.