# Quizzes for General CS I (320101) Fall 2012

Michael Kohlhase
Jacobs University Bremen
For Course Purposes Only

January 3, 2013

## Contents

**Quiz 1: Introductory Quiz** **2**

**Quiz 2: Math Talk and Peano Axioms** **3**

**Quiz 3: Relations and functions** **4**

**Quiz 4: SML** **5**

**Quiz 5: SML** **6**

**Quiz 6: Substitutions and Abstract Procedures** **7**

**Quiz 7: Formal Languages and Codes** **8**

# Quiz 1: Introductory Quiz(Given Sep. 10. 2012)

4pt

**Problem 1.1 (GenCS Grading)**

State the components of the overall grade of the GenCS course and discuss their intention.

8pt

**Problem 1.2 (What is an algorithm?)**

What is an algorithm? Give 3 examples of algorithms and explain them (be creative and make sure that at least two of them are not on the slides!).

# Quiz 2: Math Talk and Peano Axioms(Given Sep. 17. 2012)

3pt

**Problem 2.1 (Math Talk)**

Transliterate (turn into natural language) $\forall x.\exists S.x \in S \Leftrightarrow \neg(x \notin S)$

9pt

**Problem 2.2 (Natural numbers)**

Prove or refute that $s(s(o))$ and $s(s(s(o)))$ are unary natural numbers and that their successors are different.

# Quiz 3: Relations and functions(Given Sep. 24. 2012)

6pt

**Problem 3.1:** Given $A := \{1, 7, 9, 6\}$, $B := \{5, 4, 8\}$ and following relations:

$$R_1 \subseteq A \times A, \quad R_1 := \{\langle 7, 9\rangle, \langle 9, 7\rangle, \langle 1, 1\rangle, \langle 1, 6\rangle, \langle 6, 1\rangle\}$$

$$R_2 \subseteq B \times B, \quad R_2 := \{\langle 8, 4\rangle, \langle 5, 5\rangle, \langle 4, 4\rangle, \langle 8, 8\rangle, \langle 8, 5\rangle, \langle 5, 4\rangle\}$$

Determine for these relations whether they are reflexive, symmetric, and transitive. If they are not, give counterexamples (i.e. examples, where the given property is violated).

6pt

**Problem 3.2 (Function Properties)**

Consider the function $f : \mathbb{R} \to \mathbb{R}$ with $f(x) = \begin{cases} x & \text{if } x \in \mathbb{Q} \\ -x+3 & \text{else} \end{cases}$

Prove or refute that function f is bijective on $\mathbb{R}$.

# Quiz 4: SML(Given Oct. 1. 2012)

### Problem 4.1 (Square the list) 6pt

Write an SML function squareList that takes an int list and returns the list with every element squared.

Example:

```
- squareList [1, 4, 3, 10];
val it = [1, 16, 9, 100] : int list;
```

---

**Note:** $x^2 = x * x$

---

### Problem 4.2 (Add elements of list) 6pt

Implement a function that given an int list outputs the sum of its elements with the following signature and example:

```
val sum = fn : int list -> int
- sum[0,3,2,5];
val it = 10 : int
```

# Quiz 5: SML(Given Oct. 1. 2012)

12pt

**Problem 5.1 (SML and ADTs)**
You are given the following SML datatype which describes an ADT for a molecule:

**datatype** num = zero | suc **of** num;
**datatype** elt = el **of** num;
**datatype** mol = one **of** elt∗num | add **of** elt∗num∗mol;

1. Write down the formal ADT definition that describes this SML datatype.
2. Given the SML expression below, determine whether it represents a valid ground constructor term for a molecule or not and give a short explanation of your answer:

   add(el(suc(zero)),suc(suc(zero)),one(suc(suc(zero)),suc(zero)))

# Quiz 6: Substitutions and Abstract Procedures(Given Oct. 15. 2012)

12pt

**Problem 6.1 (Substitutions)**
You are given the ADT

$$\langle\{\mathbb{A},\mathbb{B},\mathbb{C}\},\{[a\colon \mathbb{A}],[b\colon \mathbb{B}],[c\colon \mathbb{C}],[f\colon \mathbb{A}\to\mathbb{A}],[g\colon \mathbb{A}\times\mathbb{B}\to\mathbb{A}],[h\colon \mathbb{B}\times\mathbb{B}\to\mathbb{A}]\}\rangle$$

Which of the following mappings are valid substitutions?

$\sigma_1 := [(f(x_{\mathbb{A}}))/x_{\mathbb{A}}],[c/y_{\mathbb{C}}],[(g(x_{\mathbb{A}},b))/Z_{\mathbb{A}}]$
$\sigma_2 := [(h(a,b))/x_{\mathbb{A}}],[(g(a,b))/y_{\mathbb{A}}]$
$\sigma_3 := [(f(a,c))/x_{\mathbb{A}}],[(g(a,b))/y_{\mathbb{A}}]$
$\sigma_4 := [f^{i+1}(x_{\mathbb{A}})/f^i(x_{\mathbb{A}})]$, $i\in\mathbb{N}$ with $f^0(x_{\mathbb{A}})=x_{\mathbb{A}}$ and $f^{i+1}(x_{\mathbb{A}})=f(f^i(x_{\mathbb{A}}))$
Justify your answers.

# Quiz 7: Formal Languages and Codes(Given Nov. 05. 2012)

12pt

**Problem 7.1 (Codes)**

You are given the alphabets $A := \{\mathtt{x}, \mathtt{y}, \mathtt{z}, \mathtt{t}\}$ and $B := \{:, ;, ), (\}$, and the function $c\colon A \to B^+$, with:

$$c(x) =:)$$
$$c(y) =;)$$
$$c(z) = (:($$
$$c(t) =;))$$

1. Please encode the string "xyxytzxz" using $c$.
2. Is $c$ a character code? Please state why or give a counter-example.
3. Check whether $c$ is a prefix code. If not, explain why, and modify the codewords of $c$ such that it becomes a prefix code.
4. Check whether the extension of the original code $c$ given above is a string code. Briefly explain your reasoning (no formal proof needed).