Assignments for General CS 1 (320101)

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Contents

Quiz 1: Fundamental Notions(Given Sep. 11.)

Problem 1.1 (Keywords of General Computer Science)
Our course started with a motivation of "General Computer Science" where some fundamental notions where introduced. Name three of these fundamental notions and give for each of them a short explanation.

Solution:

- Algorithms are abstract representations of computation instructions
- Data are representations of the objects the computations act on
- Machines are representations of the devices the computations run on

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Quiz 2: Basic Math(Given Sep. 18.)

6pt

Problem 2.1: Given $A := \{1, 7, 9, 6\}$, $B := \{5, 4, 8\}$ and following relations:

$$R_1 \subseteq A \times A, \quad R_1 := \{\langle 7, 9\rangle, \langle 9, 7\rangle, \langle 1, 1\rangle, \langle 1, 6\rangle, \langle 6, 1\rangle\}$$

$$R_2 \subseteq B \times B, \quad R_2 := \{\langle 8, 4\rangle, \langle 5, 5\rangle, \langle 4, 4\rangle, \langle 8, 8\rangle, \langle 8, 5\rangle, \langle 5, 4\rangle\}$$

Determine for these relations whether they are reflexive, symmetric, and transitive. If they are not, give counterexamples (i.e. examples, where the given property is violated).

Solution:

	reflexive	symmetric	transitive
R_1	N ($\langle 7, 7\rangle \notin R_1$)	Y	N $\langle 7, 9\rangle, \langle 9, 7\rangle \in R_1$, but $\langle 7, 7\rangle \notin R_1$
R_2	Y	N ($\langle 4, 8\rangle \in R_1$, but $\langle 8, 4\rangle \notin R_2$)	Y

Quiz 3: ML basics(Given Sep. 25.)

What is the type of the following function and what does it compute? Use example arguments for your explanation.

```
fun foo f [] = true
  | foo f (h::t) = (f h) andalso (foo f t);
```

Quiz 4: ML Types(Given Oct. 2.)

Problem 4.1: Write down the type (with explicit brackets) of the following expressions

1. `([2,2],(op*,op+))`

Hint: `op+` and `op*` are the arithmetic functions "plus" and "times".

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2. `fn (x:int) => (fn (y) => x::y)`

3pt

Problem 4.2: Write down for each of the following types an appropriate SML expression

1. `((int list) * int)-> (int list)`
2. `(int -> int) -> (int -> int)`

Quiz 6: Abstract Procedure(Given Oct. 16.)

20pt

Problem 6.1: Consider the following abstract procedure on the abstract data type of natural numbers:

$$\mathcal{P} := \langle f::\mathbb{N} \to \mathbb{N}\,;\, \{f(o) \rightsquigarrow o, f(s(o)) \rightsquigarrow o, f(s(s(n_{\mathbb{N}}))) \rightsquigarrow s(f(n_{\mathbb{N}}))\}\rangle$$

1. Show the computation process for $\mathcal{P}$ on the arguments $s(s(s(o)))$ and $s(s(s(s(s(o))))))$.
2. Give the recursion relation of $\mathcal{P}$.
3. Does $\mathcal{P}$ terminate on all inputs?
4. What function is computed by $\mathcal{P}$?

Solution:

1. $f(s(s(s(o)))) \rightsquigarrow s(f(s(o))) \rightsquigarrow s(o)$, and $f(s(s(s(s(s(o)))))) \rightsquigarrow s(f(s(s(s(s(o)))))) \rightsquigarrow s(s(f(s(s(o))))) \rightsquigarrow s(s(s(f(o)))) \rightsquigarrow s($
2. The recursion relation is $\{\langle s(s(n)), n\rangle \in (\mathbb{N} \times \mathbb{N}) \,|\, n \in \mathbb{N}\}$ (or $\langle n+2, n\rangle$)
3. the abstract procedure terminates on all inputs.
4. the abstract procedure computes the function $f\colon \mathbb{N} \to \mathbb{N}$ with $2n \mapsto n$ and $2n-1 \mapsto n$.

Quiz 7: Formal Language(Given Oct. 30.)

4pt

Problem 7.1 (Language of Unary Arithmetic)
For each of the expressions below explain why they belong to or do not belong to E_{un}.

1. div(`x201`,add//,`x12`))
2. mod(`x201`,`x01`(///,`x12`))
3. sub(mul(//,div(`x21`,//)),//)
4. add(`x`//,div(`2`//,//)),//)

Solution:

Quiz 8: Induced Boolean Function(Given Nov. 6.)

10pt

Problem 8.1 (Induced Boolean Function)
Determine the Boolean function f_e induced by the Boolean expression $e := ((x1 + x2) * \overline{x1 * x3})$.
Moreover determine the CNF and DNF of f_e.

Solution:

argument	*value*	*argument*	*value*
$\langle \mathsf{F}, \mathsf{F}, \mathsf{F} \rangle$	T	$\langle \mathsf{T}, \mathsf{F}, \mathsf{F} \rangle$	T
$\langle \mathsf{F}, \mathsf{F}, \mathsf{T} \rangle$	T	$\langle \mathsf{T}, \mathsf{F}, \mathsf{T} \rangle$	T
$\langle \mathsf{F}, \mathsf{T}, \mathsf{F} \rangle$	T	$\langle \mathsf{T}, \mathsf{T}, \mathsf{F} \rangle$	F
$\langle \mathsf{F}, \mathsf{T}, \mathsf{T} \rangle$	T	$\langle \mathsf{T}, \mathsf{T}, \mathsf{T} \rangle$	T