Assignments for General CS 2 (320201)

Michael Kohlhase
Jacobs University Bremen
For Course Purposes Only

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Contents

Quiz 1: Basic Math(Given Sept. 19.)

3pt

Problem 1.1 (Greek Alpabet)
Fill in the blanks in the table of Greek letters. Note that capitalized names denote capital Greek letters.

Symbol	θ	τ	ν	ι				
Name					gamma	chi	xi	rho

Solution:

Symbol	θ	τ	ν	ι	γ	χ	ξ	ρ
Name	theta	tau	nu	iota	gamma	chi	xi	rho

3pt

Problem 1.2: About the notions of *transitive*, *reflexive*, and *function*.

1. Give the definition of transitive as well as of reflexive relations.
2. Let $A := \{1, 2, 3\}$. Provide an example or argue against existence of a total function $f : A \to A$ which is also a transitive but not a reflexive relation.

Quiz 2: SML Types(Given Sept. 26.)

Problem 2.1: Write down the type (with explicit brackets) of the following expressions

1. `([2,2],(op*,op+))`

Hint: `op+` and `op*` are the arithmetic functions "plus" and "times".

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2. `fn (x:int) => (fn (y) => x::y)`

3pt

Problem 2.2: Write down for each of the following types an appropriate SML expression

1. `((int list) * int)-> (int list)`
2. `(int -> int) -> (int -> int)`

Quiz 3: Defining Equations(Given Oct. 4.)

Problem 3.1: Figure out the functions on natural numbers for the following defining equations

$$\delta(0) = 0$$

$$\delta(s(n)) = s(s(s(\delta(n))))$$

Problem 3.2: Figure out the functions on natural numbers for the following defining equations

$$\mu(0) = 0$$

$$\mu(s(0)) = 0$$

$$\mu(s(s(n))) = s(\mu(n))$$

Quiz 4: ADT and SML datatypes(Given Oct. 10.)

Problem 4.1: Declare an SML datatype `complex` representing complex numbers and SML functions `re` and `img` where `re` returns as real numbers the real and `img` the imaginary component of the complex number. 6pt

Moreover write down the type of the constructor of `complex` as well as of the two procedures `re` and `img`.

Use SML syntax for the whole problem.

Solution:

```
datatype complex = complex of real * real;
(* val complex = fn : real * real -> complex *)

fun re(complex(x,_)) = x;
(* val re = fn : complex -> real *)

fun img(complex(_,y)) = y;
(* val img = fn : complex -> real *)
```

Problem 4.2: Translate the abstract data type given in mathmatical notation into an SML datatype 3pt

$$\langle\{\mathbb{T}\}, \{[c_1 \colon \mathbb{T}], [c_2 \colon \mathbb{T} \to \mathbb{T} \times \mathbb{T} \to \mathbb{T}]\}\rangle$$

Solution:

```
datatype T = c1 | c2 of T -> (T * T)
```

Problem 4.3: Translate the given SML datatype 3pt

```
datatype T = 0 | c1 of T -> (T * T) -> T
```

into abstract data type in mathmatical notation.

Solution: $\langle\{\mathbb{T}\}, \{[c_1 \colon \mathbb{T}], [c_2 \colon \mathbb{T} \to (\mathbb{T} \times \mathbb{T} \to \mathbb{T}) \to \mathbb{T}]\}\rangle$

Quiz 5: Abstract Procedures(Given Oct. 17.)

4pt

Problem 5.1: Explain the concept of a "call-by-value" programming language in terms of evaluation order. Give an example program where this effects evaluation and termination, explain it.

Note: One point each for the definition, the program and the explanation.

Solution: A "call-by-value" programming language is one, where the arguments are all evaluated before the defining equations for the function are applied. As a consequence, an argument that contains a non-terminating call will be evaluated, even if the function ultimately disregards it. For instance, evaluation of the last line does not terminate.

```
fun myif (true,A,_) = A | myif (false,_,B) = B
fun bomb (n) = bomb(n+1)
myif(true,1,bomb(1))
```

2pt

Problem 5.2: Give an example of an abstract procedure that diverges on all arguments, and another one that terminates on some and diverges on others, each example with a short explanation.

Solution: The abstract procedure $\langle f{::}\mathbb{N} \to \mathbb{N}\,;\, \{f(n_\mathbb{N}) \rightsquigarrow s(f(n_\mathbb{N}))\}\rangle$ diverges everywhere. The abstract procedure $\langle f{::}\mathbb{N} \to \mathbb{N}\,;\, \{f(s(s(n_\mathbb{N}))) \rightsquigarrow n_\mathbb{N}, f(s(o)) \rightsquigarrow f(s(o))\}\rangle$ terminates on all odd numbers and diverges on all even numbers.

Quiz 6: Formal Languages and Codes(Given Nov. 7.)

3pt

Problem 6.1: Given the alphabet $A = \{a, b, c\}$ and a $L := \bigcup_{i=1}^{\infty} L_i$, where $L_1 = \{\epsilon\}$ and L_{i+1} contains the strings x, xbb, acx for all $x \in L_i$.

1. Is L a formal language?
2. Which of the following strings are in L? Justify your answer

$s_1 = acbb$	$s_2 = bbac$	$s_3 = bbbac$
$s_4 = acac$	$s_5 = acacbb$	$s_6 = acbbac$

Solution: [1]

1. L is a formal language as $L_1 \in A^+$ and every step from L_i to L_{i+1} concatenates only elements from A.
2. $s_1, s_4, s_5 \in L$

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[1] EdNote: adapt the solution from indlang

Problem 6.2: Given the alphabets $A := \{x, 3\}$ and $B := \{\mathtt{7}, @, \mathtt{z}\}$. 3pt

1. Is c with $c(x) = @@@$ and $c(3) = @@\mathtt{z7}$ a character code?
2. Is the extension of c on strings over A a code on strings? Explain your answer.

Solution: [2]

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[2] EdNote: adapt the solution from isacode

Quiz 7: Boolean Expressions(Given Nov. 14.)

3pt

Problem 7.1: Is the expression $e := (\overline{x1} + x2) * (\overline{x1} + x3)$ valid, satisfiable, unsatisfiable, falsifiable? Prove at least one property.

Problem 7.2: Give a model for C_{bool}, where the following expression are theorems: $a * \overline{a}$, $a + \overline{a}$, $a * a$, $\overline{a + a}$. 10pt

Hint: Give the truth tables for the Boolean functions.

Solution: Let $\mathcal{U} := \mathbb{B}$, and $\mathcal{I}(0) = \mathsf{F}$, $\mathcal{I}(1) = \mathsf{T}$, and

$\mathcal{I}(+)$	T	F
T	F	T
F	T	F

$\mathcal{I}(*)$	T	F
T	T	T
F	T	T

$\mathcal{I}(-)$	
T	F
F	T

With this, we have the truth tables

a	$\overline{a}$	$a * \overline{a}$
T	F	T
F	T	T

a	$\overline{a}$	$a + \overline{a}$
T	F	T
F	T	T

a	$a * a$
T	T
F	T

a	$a + a$	$\overline{a + a}$
T	F	T
F	F	T

which verify that we have indeed found the desired model.

Quiz 9: Quine McClusky Algorithm(Given Nov. 28.)

Problem 9.1 (Practising Quine McCluskey)
Use the algorithm of Quine-McCluskey to determine the minimal polynomial of the following function:

$x1$	$x2$	$x3$	f
F	F	F	F
F	F	T	F
F	T	F	T
F	T	T	F
T	F	F	T
T	F	T	T
T	T	F	F
T	T	T	T

Solution:

QMC_1 :

$$\begin{aligned} M_0 &= \{\overline{x_1}\,x_2\,\overline{x_3}, x_1\,\overline{x_2}\,\overline{x_3}, x_1\,\overline{x_2}\,x_3, x_1\,x_2\,x_3\} \\ M_1 &= \{x_1\,\overline{x_2}, x_1\,x_3\} \\ P_1 &= \{\overline{x_1}\,x_2\,\overline{x_3}\} \\ M_2 &= \emptyset \\ P_2 &= \{x_1\,\overline{x_2}, x_1\,x_3\} \end{aligned}$$

QMC_2 :

	FTF	TFF	TFT	TTT
$x_1\,\overline{x_2}$	F	T	T	F
$x_1\,x_3$	F	F	T	T
$\overline{x_1}\,x_2\,\overline{x_3}$	T	F	F	F

Final result: 1. $f = x_1\,\overline{x_2} + x_1\,x_3 + \overline{x_1}\,x_2\,\overline{x_3}$
