Name: Matriculation Number:

Midterm Exam
General CS (320101)

October 27, 2015

You have 75 minutes(sharp) for the test;
Write the solutions to the sheet.

The estimated time for solving this exam is 62 minutes, leaving you 13 minutes for revising your exam.

You can reach 56 points if you solve all problems. You will only need 50 points for a perfect score, i.e. 6 points are bonus points.

Different problems test different skills and knowledge, so do not get stuck on one problem.

	To be used for grading, do not write here								
prob.	0.1	0.2	0.3	0.4	0.5	0.6	0.7	Sum	grade
total	2	8	8	7	12	7	12	56	
reached									

Please consider the following rules; otherwise you may lose points:

- Always justify your statements. Unless you are explicitly allowed to, do not just answer "yes" or "no", but instead prove your statement or refer to an appropriate definition or theorem from the lecture.
- If you write program code, give comments, so that we can award you partial credits!

1 GenCS Classics and Induction

Problem 0.1 (Greek Letters)
Fill in the blanks in the table of Greek letters. Note that capitalized names denote capital Greek letters. 2pt

2min

Symbol	Σ	ρ	ξ	δ				
Name					*sigma*	*Phi*	*omega*	*psi*

Solution:

Symbol	Σ	ρ	ξ	δ	σ	Φ	ω	ψ
Name	*Sigma*	*rho*	*xi*	*delta*	*sigma*	*Phi*	*omega*	*psi*

Problem 0.2 (Induction)
Prove by induction or refute the following equation 8pt

10min

$$\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\frac{n+1}{2n}$$

for all $n \geq 2$.

Solution:
Proof: Proof by induction

P.1.1 Base Case: Clearly the equation holds for $n = 2$. □

P.1.2 Step Case:

P.1.2.1 Assume for some n we have $\prod_{i=2}^{n}\left(1-\frac{1}{i^2}\right)=\frac{n+1}{2n}$

P.1.2.2 Proof for $n+1$: $\prod_{i=2}^{n+1}\left(1-\frac{1}{i^2}\right) = \left(1-\frac{1}{(n+1)^2}\right)\cdot\prod_{i=2}^{1}\left(1-\frac{1}{i^2}\right) = \frac{(n+1)^2-1}{(n+1)^2}\cdot\frac{n+1}{2n} = \frac{(n+1)^2-1}{(n+1)2n} = \frac{n^2+2n}{2n(n+1)} = \frac{n+2}{2(n+1)}$ □

□

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2 MathTalk

Problem 0.3 (Mathtalk)

8pt

8min

1. Write down the set of even numbers between 1 and 999 using mathtalk.
2. Give a definition of the set of prime numbers using mathtalk
3. Prove from the definition of $\cap$ or refute by giving a counter example

$$A\cap(B\cup C)=(A\cup B)\cap C$$

4. Let $A = \{1, 2, 3, 4, 5\}$. Which elements are in the set $\{x \in A \,|\, x \leq 4 \Rightarrow x^2 < \#(A)\}$

Solution:

1. $\{x \in \mathbb{N} \,|\, 1 \leq x \leq 999 \wedge (\exists y \in \mathbb{N}. x = 2y)\}$
2. $\{x \in \mathbb{N} \,|\, x > 1 \wedge (\nexists a \in \mathbb{N}. \exists b \in \mathbb{N}. 1 < a < x \wedge 1 < b < x \wedge x = ab)\}$
3. Counter example: Let $A = \emptyset$, $B = \{1\}$, $C = \{1\}$. Then $A \cap (B \cup C) = \emptyset$, while $(A \cup B) \cap C = \{1\}$.
4. $\{1, 2, 5\}$

3 Relations and Functions

Problem 0.4 (Relations)

Let $A = \{a, b, c\}$. 7pt 7min

1. Is the empty relation $\emptyset$ symmetric on A?
2. Write down three transitive relations on A containing 2, 3, and 4 elements respectively.
3. What is the maximal size of a partial order on A?

Solution:

1. Yes, of course
2. Possible choices would be
   - $\{(a, a), (b, b)\}$
   - $\{(a, b), (b, c), (a, c)\}$
   - $\{(a, a), (a, b), (b, a), (b, b)\}$
3. It's 6. We have $\#(A \times A) = 9$. Six of those pairs are of the form (x, y), where $x \neq y$. Since we have to be antisymmetric, at most 3 of them can be in the relation. It's easy to find such a relation containing 6 elements.

4 Abstract Data Types and Abstract Procedures

Problem 0.5 (ADT for family tree)

12pt 15min

Design an abstract data type to represent a family tree. Restrict the tree to parent relationships, i.e. if you have a person A, the tree could contain A's parents, the parents of A's parents, and so on. Keep in mind that we won't know all the ancestors, so you should find a way to denote that someone's parents are unknown.

Every person shall be associated with an age. You may assume that an abstract data type for natural numbers and a procedure for adding them are given.

Provide an example term for a tree with at least 4 persons.

Define an abstract procedure that returns the total age of all persons in a tree.

Solution:

5 Programming in Standard ML

Problem 0.6 (Collatz Sequence)

Let 7pt

$$f(n) := \begin{cases} \frac{n}{2} & \text{if } n \text{ is even} \\ 3n+1 & \text{else} \end{cases}$$

7min

The sequence $x, f(x), f(f(x)), \ldots$ is called the Collatz sequence; It eventually reaches 1. Write an SML function `collatz` that takes an integer x and returns a list with the elements of the corresponding Collatz sequence before the first 1.

Hint: In SML an integer x is even iff `x mod 2 = 0`.

Solution:

```
local
  fun f(n) = if n mod 2 = 0 then n div 2 else 3*n + 1;
in
  fun collatz(1) = nil
    | collatz(n) = n::collatz(f(n));
end
```

Problem 0.7 (Permutations)

Write an SML function `is_permutation` that takes two lists and returns `true` if and only if one list is a permutation of the other one. A permutation is a reordering. 12pt

13min

Example:

```
  is_permutation([15, 32, 1], [13, 3]) = false;
  is_permutation([15, 32, 1], [1, 32, 15]) = true;
  is_permutation([#"a", #"b"], [#"b", #"c"]) = false;
  is_permutation([#"a", #"b", #"c"], [#"a", #"c", #"b"]) = true;
```

Solution:

```
fun contains(a, h::t) = if a = h then true else contains(a, t)
  | contains(_, nil) = false;

fun remove(a, h::t) = if a = h then t else h::remove(a, t)
  | remove(a, nil) = raise ThisShouldntHappen;


fun is_permutation(h::t, l) = if contains(h, l) then
          is_permutation(t, remove(h, l)) else false
  | is_permutation(nil, nil) = true
  | is_permutation(nil, _) = false;
```