Name: Matriculation Number:

Final Exam
General CS (320101)

December 12., 2015

You have two hours(sharp) for the test;
Write the solutions to the sheet.

The estimated time for solving this exam is 110 minutes, leaving you 10 minutes for revising your exam.

You can reach 110 points if you solve all problems. You will only need 100 points for a perfect score, i.e. 10 points are bonus points.

Different problems test different skills and knowledge, so do not get stuck on one problem.

	To be used for grading, do not write here													
prob.	1.1	2.1	3.1	3.2	4.1	4.2	5.1	6.1	7.1	7.2	8.1	8.2	Sum	grade
total	5	10	8	5	15	10	15	10	6	8	8	10	110	
reached														

Please consider the following rules; otherwise you may lose points:

- "Prove or refute" means: If you think that the statement is correct, give a formal proof. If not, give a counter-example that makes it fail.
- Always justify your statements. Unless you are explicitly allowed to, do not just answer "yes" or "no", but instead prove your statement or refer to an appropriate definition or theorem from the lecture.
- If you write program code, give comments!

1 Easy Points

Problem 1.1 (Greek Letters)

Fill in the blanks in the following table of Greek letters. Note that capitalized names denote capital Greek letters. 5pt 3min

Symbol		Ξ	η			Λ		ω	I	
Name	delta			sigma	Psi		Omega			chi

Solution:

Symbol	δ	Ξ	η	σ	Ψ	Λ	Ω	ω	I	χ
Name	delta	Xi	eta	sigma	Psi	Lambda	Omega	omega	Iota	chi

2 Induction

Problem 2.1 (Proof by Induction)

Consider the following recursively defined function: 10pt 10min

- $F(0) = 0$
- $F(1) = 1$
- $F(n) = F(n-1) + F(n-2)$ for all $n \geq 2$.

Prove by induction or refute that

$$F(n) \leq \left(\frac{1+\sqrt{5}}{2}\right)^n$$

Hint:

$$\frac{1+\sqrt{5}}{2} + 1 = \frac{3+\sqrt{5}}{2} = \frac{1+2\sqrt{5}+5}{4} = \left(\frac{1+\sqrt{5}}{2}\right)^2$$

3 Relations and Functions

Problem 3.1 (Relation Properties)

Let $A := \{7, 21, 14, 3\}$, consider the following relations on A: 8pt 10min

- $R_1 := \{(14,14), (21,3), (3,14), (21,14), (14,3), (3,21), (14,21), (7,7)\}$
- $R_2 := \{(21,3), (7,3), (21,21), (7,7), (7,14), (3,3), (14,14)\}$
- $R_3 := \{(7,7), (21,21), (3,3), (14,14)\}$

1. Which of the relations are reflexive, symmetric and transitive?
2. Are R_1, R_2 and R_3 functions?
3. Give example of a linear partial order on A.

Justify your answers!

Problem 3.2 (Function Definition)

Let A and B be sets. State the definition of the concept of a partial function with domain A and codomain B. Also state the definition of a total function with domain A and codomain B. 5pt 5min

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Solution: Let A and B be sets, then a relation $R \subseteq AB$ is called a **partial/total function, iff for each $a \in A$, there is at most/exactly one $b \in B$, such that $(a, b) \in R$.**

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4 SML

Problem 4.1 (Substitutions)

Given the following SML datatypes 15pt 15min

```
datatype BoolExp = Var of string | True | False | Not of BoolExp |
                   And of BoolExp∗BoolExp | Or of BoolExp∗BoolExp;

datatype Substitution = Subst of string ∗ BoolExp;
```

Write an SML function substitute that takes a list of substitutions and a boolean expression and applies the substitutions to the boolean expression (in the same order as they are listed).

Example:

```
substitute(Subst("x1", True)::Subst("x2", Not(Var("x3")))::nil,
           And(Or(Not(Var("x1")), Var("x2")), Not(And(Var("x3"), Var("x1")))))
= And(Or(Not(True), Not(Var("x3"))), Not(And(Var("x3"), True)));
```

Solution:

```
fun subst(Subst(str, expr), Var(v)) = if str=v then expr else Var(v)
  | subst(_, True) = True
  | subst(_, False) = False
  | subst(s, Not(a)) = Not(subst(s, a))
  | subst(s, And(a, b)) = And(subst(s, a), subst(s, b))
  | subst(s, Or(a, b)) = Or(subst(s, a), subst(s, b));

fun substitute(nil, expr) = expr
  | substitute(h::t, expr) = substitute(t, subst(h, expr));
```

Problem 4.2 (Duplicates)

Write an SML function that removes all duplicate elements from a list. For instance 10pt 10min

```
remove_duplicates([true, true, false]) = [true, false];
remove_duplicates([5,3,12,3,3,2]) = [5,3,12,2];
```

Hint: Write a helper function that removes duplicates, but remembers what it has already found in an argument.

Solution:

```
fun member(a, h::t) = a=h orelse member(a, t)
  | member(_, nil) = false;

fun helper(found_already, h::t) =
if member(h, found_already) then
   helper(found_already, t)
else
h::helper(h::found_already, t)
  | helper(_, nil) = nil;

fun remove_duplicates(l) = helper(nil, l);
```

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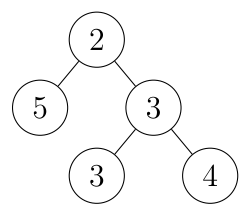
5 Abstract Datatypes

Problem 5.1 (Proper Binary Trees)

15pt

15min

1. Design an abstract data type for proper binary trees storing unary natural numbers. In a proper binary tree, every node other than the leaves has exactly two children.
2. Give the representation of the binary tree on the right in your ADT.
3. A binary tree has the min-heap property if the value of each node is greater than or equal to the value of its parent. Create an abstract procedure that checks if a proper binary tree has the min-heap property.



Note: You may define as many helper procedures as you need.
You cannot assume that binary numbers are defined already.

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6 Formal Languages

Problem 6.1 (Codes)

Let $A = \{0, 1, 2\}$ and $B = \{0, 1\}$ be alphabets. 10pt

10min

1. Specify a prefix code $p : A \to B^+$
2. Apply the extension of p to the string 02201.
3. Specify a character code $c : A \to B^+$ such that
   (a) c is not a prefix code
   (b) The extension of c is a string code, i.e. it is injective.
4. Apply the extension of c to the string 02201.

7 Boolean Algebra

Problem 7.1 (Boolean Expression)

Given the Boolean expression 6pt

6min

$$x_1 * (x_2 + x_3 * x_0) + x_1 * (x_0 * x_3)$$

1. What are the cost and depth of the expression?
2. Find an equivalent expression with smaller cost. What is the new cost?
3. Find an equivalent expression with smaller depth. What is the new depth?

Solution:

Problem 7.2 (QMC Algorithm)

Execute the Quine-McCluskey algorithm to get the minimum polynomial for the Boolean function given by 8pt

8min

x_1	x_2	x_3	x_4	f
T	T	T	T	T
T	T	T	F	F
T	T	F	T	T
T	T	F	F	F
T	F	T	T	T
T	F	T	F	F
T	F	F	T	T
T	F	F	F	F
F	T	T	T	T
F	T	T	F	F
F	T	F	T	F
F	T	F	F	F
F	F	T	T	T
F	F	T	F	F
F	F	F	T	F
F	F	F	F	F

Solution:

QMC_1 :

x_1	x_2	x_3	x_4
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	T
F	F	T	T

x_1	x_2	x_3	x_4
T	T	X	T
T	X	T	T
X	T	T	T
T	X	F	T
T	F	X	T
X	F	T	T
F	X	T	T

x_1	x_2	x_3	x_4
T	X	X	T
X	X	T	T

Therefore the prime implicants are $x_1\, x_4$ and $x_3\, x_4$

QMC_2 :

	TTTT	TTFT	TFTT	TFFT	FTTT	FFTT
$x_1\, x_4$	T	T	T	T	F	F
$x_3\, x_4$	T	F	T	F	T	T

Therefore both prime implicants are essential.

Final result: $f = x_1\, x_4 + x_3\, x_4$

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8 Propositional Logic

Problem 8.1 (Hilbert Axioms)

Proove the K and S axioms of the Hilbert calculus using the tableau method. 8pt

8min

Recall the definitions of K and S:

1. $K := P \Rightarrow Q \Rightarrow P$
2. $S := (P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R$

Hint: You can use the derived rules

Problem 8.2 (CopyLeft)

Briefly state the the copyleft clause in the GNU Public License or in the Creative Commons licenses, and explain how it works. 10pt

10min

Solution: The copyleft clause states that if a derived work of a licensed work is distributed, then it has to be licensed in exactly the same license as the licensed work.

This makes sure that anybody who wants to make a derived work of the licensed work, they have to decide whether they

- want to distribute it – then they have to license it under the copyleft, and contribute to the Open Source Domain, or
- don't, then they do not have to license it at all (but do not get the benefits of distribution/sale).
