General Computer Science (CH08-320101) Fall 2016
Assignment 10: Bonus Homework
– Given Nov. 30., Due Dec. 7. –

Problem 10.1 (Course Evaluation (Bonus))
Please remember to complete the course evaluation. If you submit a confirmation that you have completed the evaluation to JGrader – a screenshot of the "courses to evaluate" page will do, you will receive 10 bonus points for this course. 10pt

Problem 10.2 (Bernoulli inequality)
Prove by induction the Bernoulli inequality: 10pt

$$(1+x)^n \geq nx$$

where $n \in \mathbb{N}$, $x \in \mathbb{Q}$, and $x \geq -1$

Hint: You can acomplish this by proving a stronger statement first, namely that the left hand side is greater or equal to $nx+1$.

Problem 10.3 Develop an abstract data type for dates in the Gregorian calendar, i.e. the calendar in use in Europe with dates of the form "November 21. 2003" (you may disregard the BC ("before Christ") dates and leap years, but not differing month lengths). Give the defining equations for 10pt

1. computing the number of days between two dates
2. deciding the order of two dates.

Problem 10.4 (Towers of Hanoi)
Given a stack of n disks arranged from largest on the bottom to smallest on top placed on a rod, together with two empty rods, the towers of Hanoi puzzle asks for the minimum number of moves required to move the stack from one rod to another, where moves are allowed only if they place smaller disks on top of larger disks. 10pt

Write an SML function hanoi that takes the number of discs and returns a list of pair of moves between the stacks that solves our problem.

```
val hanoi = fn : int -> (int * int) list

- hanoi(1);
val it = [(1,3)] : (int * int) list

- hanoi(2);
val it = [(1,2),(1,3),(2,3)] : (int * int) list
```

Problem 10.5 (Binary Language)
Given the alphabet $A = \{0, 1\}$ for binary numbers consider the following tasks: 10pt

1. Define a formal language such that all the strings in it are binary numbers whose equivalent natural number is of the form $8 \cdot n + 1$, where $n \in \mathbb{N}$. First consider the simplest, most intuitive solution that comes to mind.

2. Now consider c as being the mapping between the binary numbers with the property given above and natural numbers. Is this a character code? Is it a prefix code? Justify your answer.
3. Now consider that you are also required to be space-efficient, so design a formal language such that you do not create duplicate strings in the process..

Problem 10.6 Use the algorithm of Quine-McCluskey and explain the intermediate steps to determine the minimal polynomial of the following function: 10pt

$x1$	$x2$	$x3$	$x4$	f
F	F	F	F	F
F	F	F	T	F
F	F	T	F	T
F	F	T	T	T
F	T	F	F	T
F	T	F	T	F
F	T	T	F	F
F	T	T	T	F
T	F	F	F	T
T	F	F	T	F
T	F	T	F	T
T	F	T	T	T
T	T	F	F	T
T	T	F	T	F
T	T	T	F	F
T	T	T	T	F

Problem 10.7 (A Hilbert Calculus)

Consider the Hilbert-style calculus given by the axioms 10pt

1. $K := P \Rightarrow Q \Rightarrow P$
2. $S := (P \Rightarrow Q \Rightarrow R) \Rightarrow (P \Rightarrow Q) \Rightarrow P \Rightarrow R$

and the rules:

1. $\dfrac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{A}}{\mathbf{B}}\,\mathrm{MP}$
2. $\dfrac{\mathbf{A}}{[\mathbf{B}/X](\mathbf{A})}\,\mathrm{Subst}$

Prove that $((\mathbf{A} \Rightarrow \mathbf{B} \Rightarrow \mathbf{C}) \Rightarrow (\mathbf{A} \Rightarrow \mathbf{B})) \Rightarrow (\mathbf{A} \Rightarrow \mathbf{B} \Rightarrow \mathbf{C}) \Rightarrow \mathbf{A} \Rightarrow \mathbf{C}$.

Hint: Look at the given rules and find out which one is better suited for starting the proof.

Problem 10.8 (Natural Deduction)

10pt

Given the following inference rules for $\mathcal{ND}^0$:

Introduction Elimination Implication

$$\frac{\mathbf{A} \quad \mathbf{B}}{\mathbf{A} \wedge \mathbf{B}} \wedge I \qquad \frac{\mathbf{A} \wedge \mathbf{B}}{\mathbf{A}} \wedge E_l \quad \frac{\mathbf{A} \wedge \mathbf{B}}{\mathbf{B}} \wedge E_r \qquad \frac{\genfrac{}{}{0pt}{}{[\mathbf{A}]^1}{\overline{\overline{\mathbf{B}}}}}{\mathbf{A} \Rightarrow \mathbf{B}} \Rightarrow I^1$$

Prove that $\mathbf{A} \wedge (\mathbf{B} \wedge \mathbf{C}) \Rightarrow \mathbf{C} \wedge \mathbf{A}$. Specify the rules applied at each step.

Problem 10.9 (Refutation and model generation in Tableau Calculus)

10pt

1. Prove the following proposition:

$$\models \neg A \wedge \neg B \Rightarrow \neg (A \vee B)$$

2. Find all models for the following proposition:

$$\models (A \Rightarrow B) \wedge (B \Rightarrow A \wedge B)$$

Hint: You may use derived rules for implication and disjunction.
