Midterm Exam
General CS 1 (320101)

October 21, 2008

LAST NAME(s):

FIRST NAME(s):

MATRICULATION NUMBER:

You have one hour (sharp) for the test;
Write the solutions to the sheet.

You can reach 29 points if you solve all problems. You will only need 25 points for a perfect score, i. e. 4 points are bonus points.

You have ample time (60 minutes), so take it slow and avoid rushing to mistakes!

Different problems test different skills and knowledge, so do not get stuck on one problem.

To be used for grading, do not write into this box										
prob.	1.1	1.2	1.3	1.4	2.1	2.2	3.1	3.2	Sum	grade
total	2	2	3	4	3	5	6	4	29	
reached										

1 Mathematical Foundations

2pt

Problem 1.1 (Greek Alpabet)
Fill in the blanks in the table of Greek letters. Note that capitalized names denote capital Greek letters.

Symbol	θ	τ	ν	ι				
Name					gamma	chi	xi	rho

2pt

Problem 1.2 (Properties of Sets)
Prove that:

1. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
2. $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Use MathTalk throughout the proof.

Problem 1.3 (Sets and Functions) 3pt

Let A and B be sets such that:

- A: $\forall S.S \textit{ is a set} \Rightarrow (A \subseteq S)$
- B: $B = \mathcal{P}(A)$

and let $f\colon A \to B$ be a total injective function from A to B.
Your task is to:

1. state when a function $f\colon A \to B$ is called injective. Use math-talk.
2. identify A and B.
3. give an example for f or explain why f does not exist.

Problem 1.4 (Bernoulli inequality) 4pt

Prove by induction the Bernoulli inequality:

$$(1+x)^n \geq nx$$

where $n \in \mathbb{N}$, $x \in \mathbb{Q}$, and $x \geq -1$

Hint: You can acomplish this by proving a stronger statement first, namely that the left hand side is greater or equal to $nx + 1$.

2 Abstract Data Types and Abstract Procedures

3pt

Problem 2.1 (ADT for trains)
Write an ADT for train configurations. Each train has a locomotive (engine) in the front and a number of cars attached to it. Each car can be either a passenger car or a cargo car. Cargo cars are characterized by capacity which can be either 'small' or 'large'.

Using your representation of trains, write down a train with three passenger cars after the engine and two cargo cars at the end, having large and small capacity, respectively.

Problem 2.2 (Abstract Procedures) 5pt

Given the ADT for natural numbers

$$\langle \{\mathbb{N}\}, \{[o\colon \mathbb{N}], [s\colon \mathbb{N} \to \mathbb{N}]\} \rangle$$

and the following procedures:
$\langle f :: \mathbb{N} \times \mathbb{N} \rightsquigarrow \mathbb{N}\,;\, \{f(o,o) \rightsquigarrow o, f(o,y) \rightsquigarrow g(o,y), f(s(x),y) \rightsquigarrow s(g(x,s(y)))\}\rangle$
$\langle g :: \mathbb{N} \times \mathbb{N} \rightsquigarrow \mathbb{N}\,;\, \{g(o,o) \rightsquigarrow o, g(x,o) \rightsquigarrow f(x,o), g(x,s(y)) \rightsquigarrow s(s(f(x,y)))\}\rangle$

1. Show the computation process for:
   $f(s(s(o)), o)$
   and
   $f(s(o), s(s(o)))$

2. What arithmetic expression does f compute and what arithmetic expression does g compute?

3. Do f and g terminate for all inputs?

3 Programming in Standard ML

6pt

Problem 3.1 (Frequency of characters in a list)
Write an SML function that given a string returns the frequency of characters in that string. The signature of the function is `fn : string -> (char * int) list`

ex:

```
freq "Red Riding Hood";

val it = [(#"R",2),(#"e",1),(#"d",3),(#" ",2),(#"i",2),(#"n",1),
          (#"g",1),(#"H",1), (#"o",2)] : (char * int) list
```

Problem 3.2 (Find My Children) 4pt

Suppose you have 2 lists given, the first one contains husband-wife pairs, the second one contains mother-child pairs. Write an SML function `FatherChildren` that returns a list of all children for a given father, or `nil` if the father has no children yet. If the father is not in the list, you raise a `NoFather` exception. Assume that there are no two fathers with the same name, and there are no two mothers with the same name.

The signature of the function is

```
fn : string * (string * string) list * (string * string) list -> string list
```

ex:

```
val x = [("Brad", "Angelina"), ("Ramratan", "Shashi"), ("Dragi", "Vesna")];
val y = [("Angelina", "Shiloh"),("Angelina", "Knox"), ("Angelina", "Vivienne")
        , ("Shashi", "Richa"),("Vesna", "Pavlinka")];

FatherChildren("Brad", x, y);
val it = ["Shiloh","Knox","Vivienne"] : string list
```

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