Midterm Exam
General CS 1 (320101)
October 23. 2006

NAME:

MATRICULATION NUMBER:

You have one hour (sharp) for the test;
Write the solutions to the sheet.

You can reach 57 points if you solve all problems. You will only need 55 points for a perfect score, i.e. two points are bonus points.

You have ample time, so take it slow and avoid rushing to mistakes!

Different problems test different skills and knowledge, so do not get stuck on one problem.

To be used for grading, do not write into this box										
prob.	1.1	1.2	1.3	2	3.1	3.2	4.1	4.2	Sum	grade
total	4	6	7	8	4	10	6	12	57	
reached										

1 Elementary Discrete Mathematics

4pt
2min

Problem 1.1 (Greek Letters)
Fill in the blanks in the table of Greek letters. Note that capitalized names denote capital Greek letters.

Symbol	Σ	ρ	ξ	δ				
Name					*sigma*	*Phi*	*omega*	*psi*

6pt
6min

Problem 1.2: Let R and S be (non empty) relations on some given set A. Prove or refute each of the three statements

1. If R and S are symmetric then $R \cap S$ is symmetric
2. If R is reflexive then all subsets of R are reflexive
3. If R is transitive then R^{-1} is transitive

7pt
10min

Problem 1.3: Using induction, show that the sum of the first n odd numbers, equals n^2, i.e.:

$$\sum_{i=1}^{n} 2i - 1 = n^2$$

2 Substitution

8pt
8min

Problem 2.1: Apply the substitutions $\sigma := [h(a, f(a), b)/x], [g(c)/y]$ and $\tau := [f(y)/x], [g(z)/y], [x/z]$ to the terms $s := g(x, h(y), z)$ and $t := h(g(x, y, g(a, y, x)))$

Note: We don't care about the type in this problem, instead we assume that all symbols are appropriately typed.

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3 Abstract Data Types and Abstract Procedures

6pt
6min

Problem 3.1 (SML datatypes vs Abstract Data Types)
Given the SML datatypes

1. `datatype A = a | f of A * A`
2. `datatype B = g of (A * B) -> B`

Write down one abstract data type in math notation representing both SML datatypes at once.

12pt
12min

Problem 3.2 (Mixed Abstract Procedures)
Consider the following mixed abstract procedures on the abstract data type of natural numbers:

$$\mathcal{F} := (\langle f :: \mathbb{N} \rightsquigarrow \mathbb{N}\,;\, (\{f(0) \rightsquigarrow 0, f(s(0)) \rightsquigarrow g(s(s(0))), f(s(s(n))) \rightsquigarrow g(s(n))\})\rangle)$$

$$\mathcal{G} := (\langle g :: \mathbb{N} \rightsquigarrow \mathbb{N}\,;\, (\{g(0) \rightsquigarrow 0, g(s(n)) \rightsquigarrow f(n)\})\rangle)$$

1. Show the computation process of $f(termappss(s(s(0))))$.
2. Do they terminate on all inputs? Justify your answer!

4 Programming in Standard ML

6pt
6min

Problem 4.1 (Call by Value)
Explain the concept of a "call-by-value" programming language in terms of evaluation order. Give an example program where this affects evaluation and termination, explain it.

12pt
12min

Problem 4.2 (Filter and Unique Element)
Write two functions `filter` and `that` in SML that take a predicate p (a function with result type `bool`) and a list l, where

- `filter` returns the list of all members a of l where $p(a)$ evaluates to `true`.
- `that` returns a if there is exactly one a in l such that $p(a)$ evaluates to `true` and raises the exception `NotUnique` if there are two or more such a, and the exception `NotExistent`, if p evaluates to `false` on all members of l.