Midterm Exam
General CS 1 (320101)
October 25. 2005

NAME:

MATRICULATION NUMBER:

You have one hour (sharp) for the test;
Write the solutions to the sheet.

You can reach 61 points if you solve all problems. You will only need 58 points for a perfect score, i.e. three points are bonus points.

You have ample time, so take it slow and avoid rushing to mistakes!

Different problems test different skills and knowledge, so do not get stuck on one problem.

To be used for grading, do not write into this box										
prob.	1.1	1.2	1.3	1.4	2.1	3.1	3.2	4.1	Sum	grade
total	3	5	10	10	8	4	6	15	61	
reached										

1 Elementary Discrete Mathematics

Problem 1.1 (Greek Letters) 3pt 3min

Fill in the blanks in the table of Greek letters. Note that capitalized names denote capital Greek letters.

Symbol	Σ	ρ	ξ	δ				
Name					*sigma*	*Phi*	*omega*	*psi*

Problem 1.2 (Function Definition) 5pt 5min

Let A and B be sets. State the definition of the concept of a partial function with domain A and codomain B. Also state the definition of a total function with domain A and codomain B.

Problem 1.3 (Asymmetry) 10pt 10min

Define

- We call a relation R **irreflexive** iff $\forall a \in A.\langle a, a\rangle \notin R$.
- We call a relation R **asymmetric** iff $\forall a, b \in A.\langle a, b\rangle \in R \Rightarrow \langle b, a\rangle \notin R$.

Prove that any irreflexive and transitive relation is also asymmetric.

Problem 1.4 (Induction) 10pt 10min

Prove by induction or refute that for all natural numbers n the following assertion holds: $n^3 + 5n$ is divisible by 6.

2 Substitution

Problem 2.1 (Substitution Applications) 8pt 8min

Let $\sigma := [h(c)/x], [g(a, f(a), b)/z]$ and $\tau := [a/x], [h(b)/y], [c/z]$ be substitutions and $s := g(x, h(y), z)$ and $t := h(g(x, y, g(a, y, x)))$ constructor terms.

1. Give an abstract data type that makes these terms and substitutions well-sorted.
2. Give the 4 result terms of substitution application σs, σt, τs, and τt).

3 Abstract Data Types and Abstract Procedures

Problem 3.1 (SML datatypes vs Abstract Data Types) 4pt 4min

Given the SML datatypes

1. `datatype A = a | f of A * A`
2. `datatype B = b | g of A -> B`

Write down one abstract data type in math notation representing both SML datatypes at once.

Problem 3.2 (Ground Constructor Terms) 6pt 6min

Assume a, b and f, g, h are constructors where a is of sort $\mathbb{A}$ and b of sort $\mathbb{B}$ with $\mathbb{A} \neq \mathbb{B}$.

1. Write down an appropriate abstract data type $\mathcal{A}$ such that $g(f(a,b), h(g(a,b)))$ is a ground constructor term in $\mathcal{A}$.
2. And for the same $\mathcal{A}$ you found justify whether or not $f(g(a,b), h(f(a,b)))$ is a ground constructor term in $\mathcal{A}$ too.

4 Programming in Standard ML

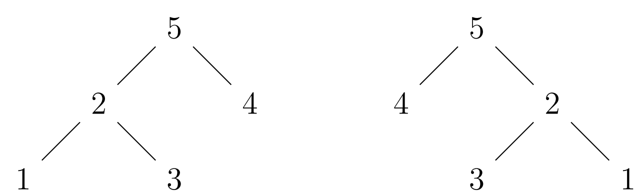
Problem 4.1 (Flip Binary Tree) 15pt 15min

A *binary tree* is a relation T on $\mathbb{N}$, such that for every $n \in \mathbb{N}$ there are two or zero $m \in \mathbb{N}$, such that $\langle n, m \rangle \in T$, and exactly one $r \in \mathbb{N}$, such that there is no $p \in \mathbb{N}$ with $\langle p, r \rangle \in T$.

We can represent the set of binary trees as the abstract data type

$$\langle \{\mathbb{T}, \mathbb{N}\}, \{[\mathit{leaf} \colon \mathbb{N} \to \mathbb{T}], [\mathit{branch} \colon \mathbb{N} \times (\mathbb{T} \times \mathbb{T}) \to \mathbb{T}]\} \rangle$$

1. Provide an corresponding SML datatype declaration `btree`.
2. construct the binary trees below within SML



3. write an SML function that takes an binary tree and returns it flipped around its vertical axis, i.e. the function transforms the left tree into the right one and the other way around.