Final Exam
General CS 1 (320101)
December 20. 2006

NAME:

MATRICULATION NUMBER:

You have two hours (sharp) for the test;
Write the solutions to the sheet.

You can reach 55 points if you solve all problems. You will only need 52 points for a perfect score, i.e. 3 points are bonus points.

You have ample time, so take it slow and avoid rushing to mistakes!

Different problems test different skills and knowledge, so do not get stuck on one problem.

To be used for grading, do not write into this box													
prob.	1.1	1.2	2.1	2.2	3.1	4.1	4.2	4.3	5.1	6.1	7.1	8.1	Sum	grade
total	3	5	6	6	6	5	3	3	4	3	6	5	55	
reached														

1 Mathematical Foundations

Problem 1.1 (Function Definition) 3pt 5min

Let A and B be sets. State the definition of the concept of a partial function with domain A and codomain B. Also state the definition of a total function with domain A and codomain B.

Problem 1.2 (Invariance of Equivalence Relations) 5pt 10min

Let A be a set and $R, S \in A^2$ be equivalence relations. Prove or refute that $R \cup S$ is an equivalence relation too.

2 Standard ML

Problem 2.1 (SML Function for Coordinate Transformation) 6pt 10min

Declare SML datatypes `cartesian` and `polar` representing points in two dimensional space in Cartesian and polar coordinates respectively.

Moreover define an SML function `cartesianToPolar : cartesian -> polar` which does the intended coordinate transformation:

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} \sqrt{\sin^2 x + cos^2 y} \\ \arctan x/y \end{pmatrix}$$

It should raise an exception for $y = 0$.

Use SML syntax for the whole problem.

Problem 2.2 (Higher-Order Functions) 6pt 15min

Write three higher-order functions that take a predicate p (a function with result type `bool`) and a list l.

- `myfilter` that returns the list of all members a of l where $p(a)$ evaluates to `true`.
- `myexists` that returns `true` if there is at least one element a in l, such that $p(a)$ evaluates to `true`.
- `myforall` that returns `true` if $p(a)$ evaluates to `true` on all elements of l.

3 Abstract Data Types and Procedures

Problem 3.1 (Abstract Procedure for Addition and Multiplication) 6pt 10min

Given the abstract data type of unary natural numbers: $\langle\{\mathbb{N}\}, \{[o\colon \mathbb{N}], [s\colon \mathbb{N} \to \mathbb{N}]\}\rangle$ Write two abstract procedures *plus* and *mult* on this abstract data type which compute the ordinary addition and multiplication respectively of unary numbers. Trace the evaluation of $mult(s(s(o)), s(s(o)))$.

4 Formal Languages and Codes

Problem 4.1 (Minimal Word Length) 5pt 10min

Let $\mathcal{A}$ and $\mathcal{B}$ alphabets with $\#\mathcal{A} > \#\mathcal{B}$. What is the minimal length of the longest codeword from $\mathcal{B}^+$ such that $c\colon \mathcal{A} \to \mathcal{B}^+$ is a character code?

Problem 4.2 (Lexical Ordering) 3pt 5min

Let $A := \{x, :, +, R, a\}$ and $\prec$ be the ordering relation on A with $a \prec R \prec + \prec : \prec x$. Order the following strings in A^* in the lexical ordering $\prec_{lex}$ induced by $\prec$.

$s_1 = RRRR$	$s_2 = RR + RRx$	$s_3 = \epsilon$
$s_4 = RR : RRa$	$s_5 = xRRRxR$	$s_6 = RRRR :$

Problem 4.3 (Character Code) 3pt 5min

Consider the character code

$$c := \{l \mapsto wi, s \mapsto \_y, \ddot{a} \mapsto w, t \mapsto ou, a \mapsto sh, T \mapsto e\_\}$$

and the mapping

$$d := ish \mapsto hol, ou \mapsto ys!, we \mapsto hap, \_y \mapsto ida, \_w \mapsto py\_.$$

Let c' be the extension of c. Write down the values of $c'(\ddot{a}Tlast)$ and $d(c'(\ddot{a}Tlast))$

5 Boolean Expressions

Problem 5.1 (Example for Validity and (Un-)Satisfiability) 4pt 8min

Given the schema $(x_1 \vee e_1) \wedge e_2$ of a Boolean expression, where e_1 and e_2 stand for arbitrary other Boolean expressions. Generate three different Boolean expressions by instantiation of e_1 and e_2 such that the result expression becomes

1. valid,
2. unsatisfiable,
3. satisfiable and falsifiable at the same time.

For the last case provide two assignments, one satisfying your expression and the other falsifying it.

6 Complexity

Problem 6.1 (Sorting Landau Sets) 3pt 10min

For two functions f and g let us define $f \sim g$ iff $f \in \Theta(g)$ and $f \prec g$ iff $f \in O(g)$. Write down in terms of these two ordering relations how the following functions are related to each other.

1. $f_1(n) := 2^{n+3}$
2. $f_2(n) := log(n^2)$
3. $f_3(n) := n^4$
4. $f_4(n) := 2^n$
5. $f_5(n) := log(n/2)$
6. $f_6(n) := 3^n$
7. $f_7(n) := (n+2)^4$

7 The Quine-McCluskey Algorithm

Problem 7.1 (Quine-McCluskey Algorithm) 6pt 10min

Use the algorithm of Quine-McCluskey to determine the minimal polynomial of the following function:

$x1$	$x2$	$x3$	f
F	F	F	T
F	F	T	F
F	T	F	F
F	T	T	T
T	F	F	F
T	F	T	F
T	T	F	T
T	T	T	T

8 Machine-Oriented Calculi

Problem 8.1 (Basics of Resolution) 5pt 10min

What are the principal steps when you try to prove the validity of a propositional formula by means of resolution calculus? In case you succeed deriving the empty clause, why does this mean you have found a proof for the validity of the initial formula?