

Midterm Exam

General CS 2 (320102)

March 31, 2008

NAME:

MATRICULATION NUMBER:

You have one hour (sharp) for the test;

Write the solutions to the sheet.

You can reach 26 points if you solve all problems. You will only need 23 points for a perfect score, i.e. three points are bonus points.

You have ample time, so take it slow and avoid rushing to mistakes!

Different problems test different skills and knowledge, so do not get stuck on one problem.

| To be used for grading, do not write into this box | | | | | | | | |
|--|-----|-----|-----|-----|-----|-----|-----|-------|
| prob. | 1.1 | 1.2 | 2.1 | 2.2 | 3.1 | 3.2 | Sum | grade |
| total | 2 | 4 | 4 | 6 | 6 | 4 | 26 | |
| reached | | | | | | | | |

1 Graphs

Problem 1.1 (Node Connectivity Relation is an Equivalence Relation)

2pt
6min

Let $G = \langle V, E \rangle$ be a *directed* graph and the relation C be defined as

$$C := \{ \langle u, v \rangle \mid \text{there is a path from } u \text{ to } v \}$$

Prove or refute that C is an equivalence relation.

4pt
10min

Problem 1.2 (Degrees in an Undirected Graph)

Prove the following assertion:

Assertion 1.1:

1. Let $G = \langle V, E \rangle$ be a *undirected* graph. Then $\sum_{i=1}^{\#V} \text{deg}(v_i)$ is an even number.
2. The number of vertices with an odd degree is even.
3. If $\#V \geq 2$ then $\exists v_1, v_2 \in V. \text{deg}(v_1) = \text{deg}(v_2)$.

Note: For undirected graphs, we introduce the notation deg with $\text{deg}(v) = \text{indeg}(v) = \text{outdeg}(v)$ for each node.

2 Combinatorial Circuits

Problem 2.1 (Binary Arithmetics)

4pt
12min

Let $A = 586$ and $B = -21$.

1. convert the numbers into a corresponding n -bit TCN system. What is the proper minimal n for converting both A and B ?
2. perform the binary operations $B + A$ and $B - A$ and check the result by converting back to the decimal system.

Problem 2.2 (Conditional circuit)

6pt
10min

Design a 2-bit conditional circuit that implements the following operation:

if $x \leq y$ then $x + y$ else $x - y$

3 Machine Programming

Problem 3.1 (Discrete Integration)

6pt
10min

Given is $n \geq 1$ stored in $D(0)$ and n numbers stored in $D(1) \dots D(n)$. Write an assembler program that calculates $\sum_{i=1}^n (-1)^i D(i)$ and stores the result in $D(1)$. Write comments to each line of your code (like in the example codes from the slides).

Problem 3.2 (Convert Highlevel Code to VM Code)

4pt
12min

Given are natural numbers i, k and s and the following piece of imperative code:

```
while(i > k) do begin
  s := s + (i - k);
  i := i - 1;
  k := k + 1;
end;
```

Suppose i , k and s is loaded on a stack correspondingly (top value being s). Convert the code into VM code, considering that the final value of s should be on the top of a stack.