General CS II (320102) Final Exam
May 23 2008

NAME:
MATRICULATION NUMBER:

You have two hours (sharp) for the test;
Write the solutions to the sheet.

You can reach 112 points if you solve all problems. You will only need 100 points for a perfect score, i. e. 12 points are bonus points.

Different problems test different skills and knowledge, so do not get stuck on one problem.

To be used for grading, do not write into this box												
prob.	1.1	2.1	3.1	4.1	5.1	6.1	7.1	8.1	8.2	9.1	Sum	grade
total	4	10	15	15	10	14	6	20	10	8	112	
reached												

1 An Old GenCS Favourite

4pt
4min

Problem 1.1 (Function Definition)
Let A and B be sets. State the definition of the concept of a partial function with domain A and codomain B. Also state the definition of a total function with domain A and codomain B.

2 Graphs

10pt
10min

Problem 2.1 (Depth of a Fully Balanced Binary Tree)
Prove that in a fully balanced binary tree with $n >= 1$ nodes, the depth is $\log_2 n$.

3 Binary numbers

15pt
15min

Problem 3.1 (Binary Numbers)

- Prove that $\langle\langle \overline{a} \rangle\rangle_n^{2s} = -\langle\langle a \rangle\rangle_n^{2s} - 1$.
- Convert -42 to 12-bit two's complement.
- Convert the hexadecimal number $BEEF$ to decimal. You do not need to carry out the computations, just write down a the answer without simplifications.

4 Combinatorial Circuits

15pt
15min

Problem 4.1 (Alarm System)

You have to devise an alarm system that signals if the image recorded by a camera changes. The camera is preprogrammed with a static image, divided into 8 regions. Whenever an observed region is different from the preprogrammed one, the corresponding input bit $(r_0, ..., r_7)$ is set to 1. The image is sampled at discrete time periods. The value of an input (clk) changes between 0 and 1 on every time interval.

Design a circuit with one output which is set to 1 if two or more regions (the inputs $(r_0, ..., r_7)$) are different from the preprogrammed image for two consecutive intervals. We do not care if different sets of regions are marked as different between the consecutive intervals. We also don't care what happens once the output is set to one.

You may use all elementary gates and all circuit blocks studied in class.

5 Virtual Machines

10pt

Problem 5.1 (Static Procedure for Binomial Coefficients) 10min

Write a $\mathcal{L}$(VM) static procedure that computes the value of the binomial $C(n,k)$. Use the recursion formula:

$$C(n+1,k+1) = C(n,k+1) + C(n,k)$$
$$C(n,0) = C(0,0) = 1$$
$$C(0,n) = 0$$

6 SML

14pt

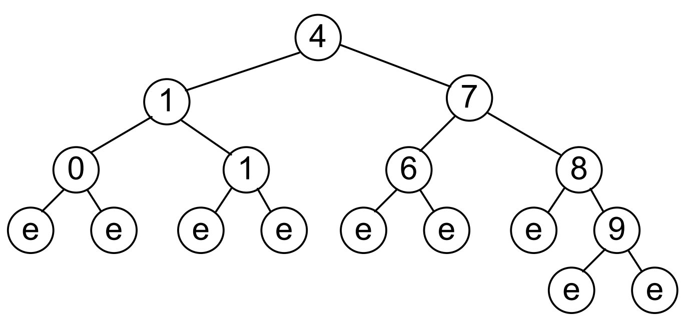
Problem 6.1 (Treesort Function) 14min

Your task is to write a treesort function in SML that sorts a list of integers by first creating a binary search tree from the list and then loading the tree (in a sorted order) back into a list.

Use the following definition of a binary search tree:

- All leaves are empty nodes.
- All internal nodes carry a value and a left and a right subtree.
- The values of all nodes in a node's left subtree are smaller than the node's value and all nodes in its right subtree are greater or equal to the node's value.

The following tree is an example of a binary search tree:



Given the following datatype:

```
datatype searchtree = empty | node of searchtree*searchtree*int;
```

The tree above would be represented as follows:

```
node(node(node(empty,empty,0),node(empty,empty,1),1),
  node(node(empty,empty,6), node(empty,node(empty,empty,9),8),7) , 4);
```

Write the functions using the searchtree datatype. The function sort should be of the following type:

```
fn treesort: int list -> int list
```

7 Turing Machines

Problem 7.1 (Turing Machine) 6pt 6min

Construct a TM that adds 1 to a binary number. Your input is preceded and followed by ##. The head points to the first # in the beginning. E.g. : ##0100## → ##0101##

Note: You will receive 6 $points - the\ number\ of\ states\ above$ 4 (excluding the halting state, if you choose to use one).

8 Problem Solving and Search

Problem 8.1 (Power Source Search) 20pt 20min

A robot is on the 5x5 map shown below. It wants to reach a power source, but its sensors only allow it to detect the source once it is in the same cell with it. Find a problem formulation in the quadruple format presented in the lecture such that depth first search will find a solution after expanding exactly 6 nodes.

Assume that the `next` function of the DFS algorithm used returns the `(action, state)` tuples in the order in which the corresponding operators are defined. For example, if your operators are `jump` and `sing`, then the next function called on state i would return a list `[(jump, state j), (sing, state k)]` and not the other way around. (this is just an example, these operators will not do a very good job ... :))

Define a path cost for this problem. What is the cost of this solution? Is the solution optimal?

How many node expansions would BFS make considering the same `next` function?

			R	
P				P

R represents the robot and P a power source.

Problem 8.2 (Greedy Search and Hill Climbing) 10pt 10min

Name *relevant* similarities and differences between greedy search and hill climbing. Describe a real-life problem where you would use greedy and one where you would use hill climbing.

Note: A search tree or the queen's problem are not real-life problems.

9 Prolog

Problem 9.1: Generate all strings with N ones and M zeros. 8pt 8min

Sample input:

```
gen(3,2,X).
X = [1, 1, 1, 0, 0] ;
X = [1, 1, 0, 1, 0] ;
X = [1, 1, 0, 0, 1] ;
X = [1, 0, 1, 1, 0] ;
X = [1, 0, 1, 0, 1] ;
X = [1, 0, 0, 1, 1] ;
X = [0, 1, 1, 1, 0] ;
X = [0, 1, 1, 0, 1] ;
X = [0, 1, 0, 1, 1] ;
X = [0, 0, 1, 1, 1] ;
```