General CS II (320102) Final Exam
May 26. 2005

NAME:
MATRICULATION NUMBER:

You have two hours (sharp) for the test;
Write the solutions to the sheet.

You can reach 109 points if you solve all problems including the bonus problem at the end. You will only need 100 points for a perfect score, i.e. 9 points are bonus points.

You have ample time, so take it slow and avoid rushing to mistakes!

Different problems test different skills and knowledge, so do not get stuck on one problem.

1 General Mathematical Foundations

Problem 1.1 (Functions)
Let A and B be sets. State the definition of a partial function with dom
codomain B.

2 Computational Logic

Problem 2.1 (Resolution Calculus)
Prove the formula $((P \vee (Q \wedge R)) \Rightarrow ((P \vee Q) \wedge (P \vee R)))$ in the resolution calculus. 10pt

Problem 2.2 (Soundness of a Calculus)
We have a calculus for propositional logic that consists of the axioms 10pt

$$(P \Rightarrow (Q \Rightarrow P)) \qquad \text{and} \qquad ((P \wedge Q) \Rightarrow (Q \wedge P))$$

and the inference rule

$$\frac{(P \vee Q) \quad \neg Q}{P}$$

Show that this calculus is sound.

3 Graphs

Problem 3.1 (Spanning Tree of a Graph) 20p

A spanning tree of a graph is a tree containing all nodes of the graph as well as a subset of its edges (but no additional edges!).

We call a graph connected, iff for any two nodes n_1 and n_2 there is a path starting at n_1 and ending at n_2.

Prove by induction that every undirected, connected, and non-empty graph has a spanning tree.

Furthermore draw the following undirected graph together with one corresponding spanning tree

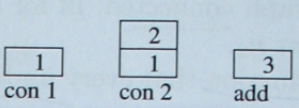
$$G = \langle \{a, b, c, d, e\}, \{\{a, b\}, \{a, c\}\{b, c\}, \{c, d\}, \{c\}, \{b, d\}, \{d, e\}\} \rangle$$

4 Virtual Machines

Problem 4.1 (Sorting Integers)

Write a $\mathcal{L}(\texttt{VM})$ program that sets two variables x and y and swaps their values if $x > y$ and leaves them unchanged otherwise. Furthermore draw the evolution of the stack during the execution for $x = 2$ and $y = 1$. 10pt

Hint: For instance the execution of $1 + 2$ can be drawn as



5 Combinational Circuits

Problem 5.1 (Half Adder)

Design an explicit combinatorial circuit for the half-adder using only $NAND$ gates. What is its cost and depth? 15pt

Hint: Recall that $((A \uparrow B) \iff \neg(A \wedge B))$ and $(A \oplus B \iff ((\neg A \wedge B) \vee (\neg B \wedge A)))$ where $\uparrow$ stands for $NAND$ and $\oplus$ for XOR.

Problem 5.2 (Arithmetic on Two's Complement Numbers (TCN)) 10pt

Compute the expressions $a + b, a - b, a' + b'$, and $a' - b'$ for

- the 8-bit TCNs $a = 01001011$, and $b = 11101100$, and
- the 9-bit TCNs $a' = 001001011$, and $b' = 111101100$

Give the result accordingly as 8-bit and 9-bit TCNs.

Hint: A transformation from TCNs to integers complicates the task unnecessarily You will be faster if you use appropriate theorems. If you do so then please cite which theorems you use.

6 Turing Machines

Problem 6.1 (Detecting 110)

Consider a Turing machine with $\{0, 1\}$ as alphabet and a tape randomly filled with ones and zeros. Find a transition table for this machine that reads the tape from left to right starting at an arbitrary position and halting as soon as it reads the sequence 110. Furthermore draw a state machine diagram representing its transition table. 10pt

7 Problem Solving and Search

Problem 7.1 (Problem Formulation) 10pt

You have three jugs, measuring 12 gallons, 8 gallons, and 3 gallons, and a water tap. You can fill the jugs up or empty them out from one to another or onto the ground. You need to measure out exactly one gallon. Give the initial state, goal test, successor function, and cost function for this problem. Choose a formulation that is precise enough to be implemented.

Note that you are not asked to find a plan that solves the measuring problem; this is almost trivial: fill the first jug, then fill the second jug from it (so there are 4 gallons left in jug 1), and then fill the third jug from jug 1; then 1 gallon is left in jug 1 as desired.

7.6

Problem 7.2: Consider a state space where the start state is the number 1 and the successor function for state n returns two states, numbers $2 * n$ and $2 * n + 1$. 10pt

1. Draw the portion of the state space for states 1 to 15.
2. Suppose the goal state is 11. List the order in which nodes will be visited for
   (a) breadth-first search,
   (b) depth-limited search with limit 3, and
   (c) iterative deepening search.