

Problem 0.1 (Solving Equations via Higher-Order Unification)

10min

Let $\alpha := (\iota \rightarrow \iota) \rightarrow \iota \rightarrow \iota$, solve the equation below via higher-order unification.

$$\lambda X_{\iota \rightarrow \iota} Y_{\iota} X(N_{\alpha} XY) = \lambda Z_{\iota \rightarrow \iota} W_{\iota} Z(Z(ZW))$$

Solution: We develop the corresponding unification problem via the higher-order transformations.

1. initial problem: $\lambda X_{\iota \rightarrow \iota} Y_{\iota} X(N_{\alpha} XY) = ? \lambda Z_{\iota \rightarrow \iota} W_{\iota} Z(Z(ZW))$
2. simplify with *SLM*: α : $a(N_{\alpha} ab) = ? a(a(ab))$ where $a \in \Sigma_{\iota \rightarrow \iota}^{S^k}$ and $b \in \Sigma_{\iota}^{S^k}$.
3. decompose with *SLM*:dec to $N_{\alpha} ab = ? a(ab)$
4. imitate *HOU*:fr to $N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(H_{\alpha} XY) \wedge N_{\alpha} ab = ? a(ab)$
5. eliminate N with *SLM*:elim to $N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(H_{\alpha} XY) \wedge a(H_{\alpha} ab) = ? a(ab)$
6. decompose with *SLM*:dec to $N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(H_{\alpha} XY) \wedge H_{\alpha} ab = ? ab$
7. imitate *HOU*:fr to $H_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(K_{\alpha} XY) \wedge N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(H_{\alpha} XY) \wedge H_{\alpha} ab = ? ab$
8. eliminate H with *SLM*:elim to

$$H_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(K_{\alpha} XY) \wedge N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(X(H_{\alpha} XY)) \wedge a(K_{\alpha} ab) = ? ab$$

9. decompose with *SLM*:dec to

$$H_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(K_{\alpha} XY) \wedge N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(X(N_{\alpha} XY)) \wedge K_{\alpha} ab = ? b$$

10. project with *HOU*:fr to

$$K_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} Y \wedge H_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(K_{\alpha} XY) \wedge N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(X(K_{\alpha} XY)) \wedge K_{\alpha} ab = ? b$$

11. eliminate K with *SLM*:elim to

$$K_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} Y \wedge H_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} XY \wedge N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(XY) \wedge b = ? b$$

12. decompose *SLM*:dec to

$$K_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} Y \wedge H_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} XY \wedge N_{\alpha} = ? \lambda X_{\iota \rightarrow \iota} Y_{\iota} X(XY)$$

13. this is a solved equational problem, so the solution is

$$[(\lambda X_{\iota \rightarrow \iota} Y_{\iota} Y)/K_{\alpha}], [(\lambda X_{\iota \rightarrow \iota} Y_{\iota} XY)/H_{\alpha}], [(\lambda X_{\iota \rightarrow \iota} Y_{\iota} X(XY))/N_{\alpha}]$$
