Name:

Birth Date:

Matriculation Number:

Field of Study:

Retake Exam
Künstliche Intelligenz 2

Feb. 19., 2018

To be used for grading, do not write here	
prob.	Sum
total	0
reached	

Exam Grade: Bonus Points: Final Grade:

Organizational Information

Please read the following directions carefully and acknowledge them with your signature.

- Bitte legen Sie Ihren Studentenausweis und einen Lichtbildausweis zur Personenkontrolle bereit!
- Die angegebene Punkteverteilung gilt unter Vorbehalt.
- Es sind keine Hilfsmittel erlaubt.
- Die Lösung einer Aufgabe muss auf den vorgesehenen freien Raum auf dem Aufgabenblatt geschrieben werden; die Rückseite des Blatts kann mitverwendet werden. Wenn der Platz nicht ausreicht, können bei der Aufsicht zusätzliche Blätter angefordert werden.
- Wenn Sie die Prüfung aus gesundheitlichen Gründen abbrechen müssen, so muss Ihre Prüfungsunfähigkeit durch eine Untersuchung in der Universitätsklinik nachgewiesen werden. Melden Sie sich in jedem Fall bei der Aufsicht und lassen Sie sich das entsprechende Formular aushändigen.
- Die Bearbeitungszeit beträgt 90 Minuten.
- Überprüfen Sie Ihr Exemplar der Klausur auf Vollständigkeit (**??** Seiten inklusive Deckblatt und Hinweise) und einwandfreies Druckbild! Vergessen Sie nicht, auf dem Deckblatt die Angaben zur Person einzutragen!

Erklärung

Durch meine Unterschrift bestätige ich den Empfang der vollständigen Klausurunterlagen und die Kenntnisnahme der obigen Informationen.

Erlangen, Feb. 19., 2018 ....

(Unterschrift)

1 Bayesian Reasoning

Problem 1.1 (AFT Tests)
Trisomy 21 (*Down syndrome*) is a genetic anomaly that can be diagnosed during pregnancy using an amniotic fluid test. 14pt

7min

The probability of a fetus having Down syndrome is strongly correllated with the age of the mother during pregnancy. For 25 year old mothers the probability is 0.08%, for 43 year old mothers it increases to 2% (we only consider those two age groups).

However, diagnostic tests are never perfect. We distinguish two kinds of errors:

- **Type I Error (False Positive)**: The test result is positive even though the child is healthy.
- **Type II Error (False Negative)**: The test result is negative even though the child has trisomy 21.

The probabilities of Type I and Type II Errors are both merely 1% for amniotic fluid tests for Down syndrome.

1. Express all the numbers given above as conditional probabilities. Use the random variable F with Domain $\{Age_{25}, Age_{43}\}$ for the age of a mother and the boolean random variables Pos and $Down$ for the propositions "*The amniotic fluid test is positive*" and "*The child has Down syndrome*" respectively.
2. Assume now we have a fixed mother of age 25 (i.e. for any event X you may assume $P(X \mid F = Age_{25}) = P(X)$). How can we compute the probability that a child has Down syndrome, given that the amniotic fluid test is positive? Give an equation that only uses the probabilities for which we have actual numbers.

Solution:

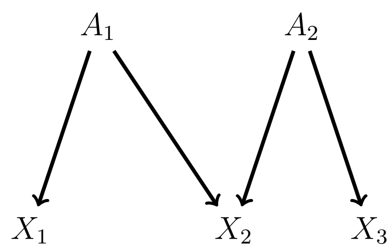
1. $P(Down \mid F = Age_{25}) = 0.0008$, $P(Down \mid F = Age_{43}) = 0.02$, $P(Pos \mid \neg Down) = 0.01$, $P(\neg Pos \mid Down) = 0.01$.
2. We normalize to $F = Age_{25}$ and compute:

$$P(Down \mid Pos) = \frac{P(Pos \mid Down) \cdot P(Down)}{P(Pos)} = \frac{P(Pos \mid Down) \cdot P(Down)}{P(Pos \wedge Down) + P(Pos \wedge \neg Down)}$$
$$= \frac{P(Pos \mid Down) \cdot P(Down)}{P(Pos \mid Down) \cdot P(Down) + P(Pos \mid \neg Down \cdot P(\neg Down))}$$
$$= \frac{(1 - P(\neg Pos \mid Down)) \cdot P(Down)}{(1 - P(\neg Pos \mid Down) \cdot P(Down)) + P(Pos \mid \neg Down) \cdot (1 - P(Down))}$$

Problem 1.2 (Bayesian Networks)

Consider the following Bayesian network with boolean variables: 16pt

8min



1. Which nodes in the network are
   - Stochastically independent
   - Conditionally independent and under which conditions?
2. What exactly (formal criterion) does an arrow between two nodes in a bayesian network mean for the associated events?

Solution:

1. - A_1 and A_2 are stochastically independent.
   - X_1 and X_3 are stoachastically independent.
   - A_1 and X_3 (respectively A_2 and X_1) are stochastically independent.
   - X_1 and X_2 are conditionally independent given A_1.
   - X_2 and X_3 are conditionally independent given A_2.
2. We draw an arrow from X_j to X_i if X_j is in the smallest set $\texttt{Parents}(X_i)$ with the property $P(X_i|X_{i-1}, ..., X_1) = P(X_i|\texttt{Parents}(X_i))$

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2 Decision Theory

Problem 2.1 (Decision Preferences)

8pt

4min

1. Name and state three of the axioms for preferences (i.e. $\prec$).
2. How are preferences related to utility functions?

Solution:

1. Orderability $(A \prec B) \vee (B \prec A) \vee (A \sim B)$

   Transitivity $(A \prec B) \wedge (B \prec C) \Rightarrow (A \prec C)$

   Continuity $A \prec B \prec C \Rightarrow \exists p([p, A; (1-p), C] \sim B)$

   Substitutability $(A \sim B) \Rightarrow ([p, A; (1-p), C] \sim [p, B; (1-p), C])$

   Monotonicity $(A \prec B) \Rightarrow (p \geq q) \Leftrightarrow ([p, A; (1-p), B] \preceq [q, A; (1-q), B])$

2. Ramsey's theorem states that given a set of preferences that obey the constraints above, there is a utility function U with $(U(A) {\geq} U(B)) \Leftrightarrow (A \preceq B)$ and.

Problem 2.2 (Expected Utility)

What is the formal(!) definition of *expected utility*? Explain every variable in the defining equation. 8pt 4min

Solution: The expected utility EU is defined as $EU(a|e) = \sum_{s'} P(R(a) = s'|a, e) \cdot U(s')$, where

1. a is the action for which we want to find out the expected utility, given the evidene e.
2. $U(s')$ is the utility of a state s'.
3. $R(a)$ is the result of the action a.

Problem 2.3 (Decision Network)

20pt

10min

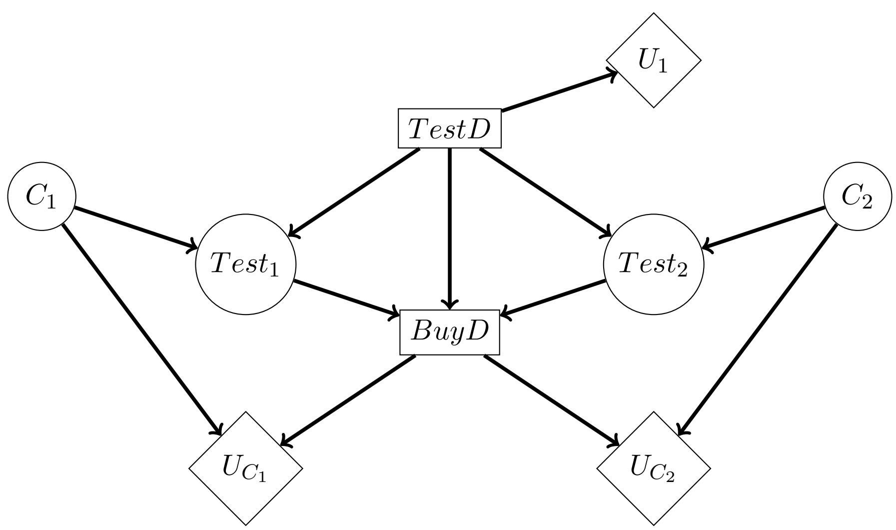
You need a new car. Your local dealership has two models on offer – C_1 for 1500$ and C_2 for 1250$. Either car can be of good quality or bad quality. If C_1 is of bad quality, repairing it will cost 200$, if C_2 is of bad quality repairing it will cost 500$. You have the choice between two tests:

1. You can either take C_1 on a short test drive for 30 minutes, which will confirm that it is of good quality with 75% probability, and that it is of bad quality (if it is) with probability 65%.
2. Or you can borrow C_2 for a whole weekend, which will confirm that it is of good quality with 80% probability, and that it is of bad quality (if it is) with probability 70%.

The a priori probability that C_1 is of good quality is 70% and for C_2 it is 65%.

1. Draw the decision network for which test to apply and which car to buy in either case (aSsume a meaningful utility function given).
2. Explain *formally* how to compute which test to apply.
3. Assume you apply test 1 and it seems like the car is of bad quality. Explain how to compute which car you should actually buy.

Solution: `http://mas.cs.umass.edu/classes/cs683/lectures-2010/Lec21_Uncertainty6-F2010-4up.pdf`

1. 

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3 Markov Models

Problem 3.1 (Bellman Equation)

State the Bellman Equation and explain 1) every symbol in the equation and 2) what the equation is used for and how. 8pt

4min

Solution:

$$U(s) = R(s) + \gamma \cdot \max_a \left(\sum_{s'} U(s') \cdot T(s, a, s') \right)$$

Problem 3.2 (Stationary)

Define what it means for a markov model to be *stationary*, and why we are interested in stationarity. 6pt 3min

Solution:

Problem 3.3 (Stock Market Predictions)

You bought SpaceY-stock recently and try to predict whether to buy more or sell. The stock market is in one of two possible states; bull state or bear state. In a bull state, it will (in the long term) be advantageous to buy stock; in a bear state it will be more advantageous to sell. 12pt 6min

If the market is in a bull state, the probability it will still be in a bull state tomorrow is 60%. If it is in a bear state, the probability it will remain so tomorrow is 80%.

If the market is in a bull state, the probability that your stock will rise that day is 90%. If it is in a bear state, your stock will more likely fall (with 60% probability).

You bought stock on day 0, and know now that back then the market was luckily in bull state. You need money and want to preferentially sell your stock, but all you can actually observe is whether your stock rises or falls on any given day.

It's now day n and you know what your stock did on every day since day 0. How do we compute the probability that the market is in a bear state right now? (using only probabilities given)

Solution: Let Bu represent a bull state and $S^{\uparrow}$ a rise in stock We have $P(Bu_0) = 1$ and

$$\langle P(Bu_d), P(\neg Bu_d)\rangle = \langle P(Bu_d|Bu_{d-1}) + P(Bu_d|\neg Bu_{d-1}), P(\neg Bu_d|Bu_{d-1}) + P(\neg Bu_d|\neg Bu_{d-1})\rangle$$

which allows us to update using the information $S_d^{\uparrow}$:

$$\langle P(Bu_d|S_d^{\uparrow}), P(\neg Bu_d|S_d^{\uparrow})\rangle = \alpha\langle P(S_d^{\uparrow}|Bu_d)P(Bu_d), P(S_d^{\uparrow}|\neg Bu_d)P(\neg Bu_d)\rangle$$

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4 Learning

Problem 4.1 (Gradient Descent)

Explain a gradient descent algorithm. 8pt

Solution: 4min

Problem 4.2 (Information Entropy)

Explain and define *information entropy*. 6pt

Solution: 3min

$$I(\langle P_1, ..., P_n \rangle) = \sum_{i=1}^{n} -P_i \log_2(P_i)$$

Problem 4.3 (Overfitting)

Explain what *overfitting* means and why we want to avoid it. 8pt

Solution: 4min

Problem 4.4 (Tennis Trees)

Consider the following decisions on whether or not to go play tennis. The target is "PlayTennis". 20pt

10min

Outlook	Temperature	Humidity	Wind	PlayTennis
Sunny	Hot	High	Weak	No
Overcast	Hot	High	Weak	Yes
Rain	Mild	High	Weak	Yes
Rain	Cool	Normal	Strong	No
Rain	Mild	Normal	Weak	Yes

Explain how to apply decision tree learning to this table.

Solution: `http://courses.cs.tamu.edu/choe/17spring/633/lectures/slide05.pdf`

$$E := I(\langle\frac{3}{5},\frac{2}{5}\rangle) = -\frac{3}{5}\log_2(\frac{3}{5}) - \frac{2}{5}\log_2(\frac{2}{5})$$

$$\begin{aligned}\texttt{Gain}(\texttt{Outlook}) &= E - \frac{1}{5}I(\langle 0,1\rangle) - \frac{1}{5}I(\langle 1,0\rangle) - \frac{3}{3}I(\langle\frac{2}{3},\frac{1}{3}\rangle)\\ \texttt{Gain}(\texttt{Temperature}) &= \ldots\end{aligned}$$

Pick whichever attribute has the highest information gain, split there and build subtrees, iterate...
