Name: Matriculation Number:

Mock Exam
Künstliche Intelligenz-1

January 9., 2017

You have one hour(sharp) for the test;
Write the solutions to the sheet.

The estimated time for solving this exam is 53 minutes, leaving you 7 minutes for revising your exam.

You can reach 106 points if you solve all problems. You will only need 100 points for a perfect score, i.e. 6 points are bonus points.

Different problems test different skills and knowledge, so do not get stuck on one problem.

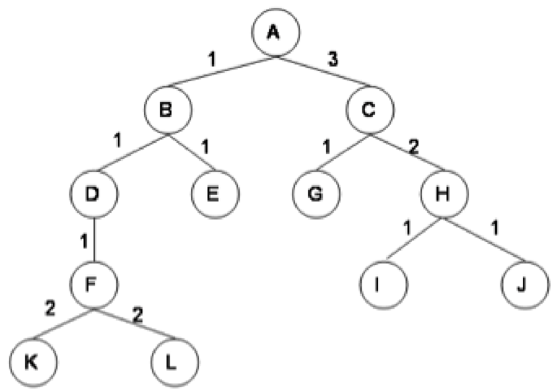
	To be used for grading, do not write here												
prob.	1.1	1.2	1.3	2.1	2.2	3.1	3.2	4.1	4.2	4.3	4.4	Sum	grade
total	8	6	6	10	10	14	26	8	6	6	6	106	
reached													

Please consider the following rules; otherwise you may lose points:

- "Prove or refute" means: If you think that the statement is correct, give a formal proof. If not, give a counter-example that makes it fail.
- Always justify your statements. Unless you are explicitly allowed to, do not just answer "yes" or "no", but instead prove your statement or refer to an appropriate definition or theorem from the lecture.
- If you write program code, give comments!

1 Search

Problem 1.1 Explain how BFS and DFS work and write down the sequences of nodes expanded by each altorithm for these algorithms. 8pt 4min



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Problem 1.2 (Admissibility limits)

The condition for a heuristic $h(n)$ to be admissible is that for all nodes n holds that $(0 \leq h(n) \leq h^*(n))$, where $h^*(n)$ is the true cost from n to goal. What happens when for all nodes, $h(n) = 0$ and when $h(n) = h^*(n)$? 6pt 3min

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Solution: When $h(n) = 0$, the search will behave like an uninformed search, and when $h(n) = h^*(n)$ the search will only expand the nodes on the optimal path to a goal.

Problem 1.3 Does a finite state space always lead to a finite search tree? How about a finite space state that is a tree? Justify your answers. 6pt 3min

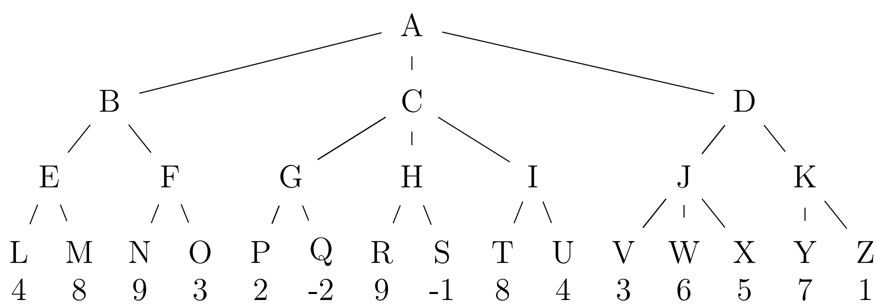
2 Adversarial Search

Problem 2.1 (Minimax Restrictions)

Name at least five criteria that a game has to satisfy in order for the minimax algorithm to be applicable. 10pt 5min

Problem 2.2 (Game Tree)

Consider the following game tree. Assume it is the maximizing player's turn to move. The values at the leaves are the static evaluation function values of the states at each of those nodes. 10pt 5min



1. Label each non-leaf node with its minimax value. See above 10 pt
2. Which move would be selected by Max? 5 pt
3. List the nodes that the alpha-beta algorithm would prune (i.e., not visit). Assume children of a node are visited left-to-right. 10 pt
4. In general (i.e., not just for the tree shown above), if we traverse a game tree by visiting children in right-to-left order instead of left-to-right, can this result in a change to 5 pt
   (a) the minimax value computed at the root?
   (b) The number of nodes pruned by the alpha-beta algorithm?

Solution:

1. A:89, B:8, C:6, E:8, F:9, G:2, H:9, I:8, J:6, K:7
2. B
3. OHRSITUKYZ
4. (a) no, (b) yes

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3 Constraint Satisfaction Problems & Inference

Problem 3.1 (CSP Heuristics)

Explain backtracking search for CSPs and the minimum remaining values (MRV) heuristic, the degree heuristic and least constraining value heuristic (LCV). 14pt 7min

Problem 3.2 (CSP: Greater (or Lesser) Chain)

Consider the general less-than chain below, which we interpret as a CSP: Each of the N variables X_i has the domain $\{1, \ldots, M\}$. The constraints between adjacent variables X_i and X_{i+1} require that $X_i < X_{i+1}$. 26pt 13min

$$X_1 < X_2 < X_3 < \cdots < X_N$$

1. For now, assume $N = M = 5$.
   (a) How many solutions does the CSP have? 1 pt
   (b) What will the domain of X_1 be after enforcing the consistency of only the arc $X_1 \to X_2$? 1 pt
   (c) What will the domain of X_1 be after enforcing the consistency of only the arcs $X_2 \to X_3$ and (then) $X_1 \to X_2$? 2 pt
   (d) What will the domain of X_1 be after fully enforcing arc consistency? 2 pt
2. Now consider the general case for arbitrary N and M.
   (a) What is the minimum number of arcs (big-O is ok) which must be processed by AC-3 (the algorithm which enforces arc consistency) on this graph before arc consistency is established? 3 pt
   (b) Imagine you wish to construct a similar family of CSPs which forces one of the two following types of solutions: either all values must be ascending or all values must be descending, from left to right. For example, if $M = N = 3$, there would be exactly two solutions: $\{1, 2, 3\}$ and $\{3, 2, 1\}$. Explain how to formulate this variant. Your answer should include a constraint graph and precise statements of variables and constraints. 4 pt

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Solution:

1. $N = M = 5$.
   (a) Just one: 1, 2, 3 ...
   (b) $\{1, 2, 3, 4\}$
   (c) $\{1, 2, 3\}$
   (d) $\{1\}$
2. (a) $O(MN)$. You will have to process many arcs multiple times, one for each domain value.

(b) Several good answers. One is have ternary constraints on each adjacent triple that the triple should be either an increasing or decreasing triple. The overlap between triples enforces that the choice be global. Another is to introduce a global variable indicating ascent or descent and have ternary constraints between adjacent nodes and the global one, allowing, for example, triples like $\langle 1, 2, < \rangle$ or $\langle 2, 1, > \rangle$.

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4 Logic

Note: For ASCII submissions, use the symbols `&`, |, `~`, `->` instead of $\wedge, \vee, \neg, \Rightarrow$.

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Problem 4.1 (Calculus Properties)

Explain briefly what the following properties of calculi mean: 8pt 4min

- correctness
- completeness

Solution:

- correctness ($\mathcal{H} \vdash \mathbf{B}$ implies $\mathcal{H} \models \mathbf{B}$) - A calculus is correct if any derivable(provable) formula is also a valid formula.
- completeness ($\mathcal{H} \models \mathbf{B}$ implies $\mathcal{H} \vdash \mathbf{B}$) - A calculus is complete if any valid formula can also be derived(proven).

Problem 4.2 (An incorrect calculus)

Why is this calculus $\mathcal{C}^2$ incorrect? 6pt 3min

- $\mathcal{C}^2$ Axiom: $\mathbf{A} \vee \neg \mathbf{A}$
- $\mathcal{C}^2$ Inference Rules: $\dfrac{\dfrac{[\mathbf{A}]^1}{\mathbf{B}}}{\mathbf{A} \Rightarrow \mathbf{B}} \Rightarrow I^1 \qquad \dfrac{\mathbf{A} \Rightarrow \mathbf{B} \quad \mathbf{B} \Rightarrow \mathbf{C}}{\mathbf{C}} \Rightarrow E$

Solution: In a correct calculus the inference rules must preserve validity. However the inference rule R2 does not preserve validity: Let $\mathbf{A} \Rightarrow \mathbf{B}$ and $\mathbf{B} \Rightarrow \mathbf{C}$ be valid. This is the case for an assignment for which $[\![\mathbf{A}]\!]_\varphi = \mathsf{F}$, $[\![\mathbf{C}]\!]_\varphi = \mathsf{F}$, and $[\![\mathbf{C}]\!]_\varphi = \mathsf{F}$. In particular the conclusion for this assignment is false: $[\![\mathbf{C}]\!]_\varphi = \mathsf{F}$. Hence the inference rule R2 does not preserve validity.

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Problem 4.3 (Resolution Calculus)

Use the resolution calculus to prove the validity of the expression: 6pt 3min

$$(X \wedge Y) \vee (X \wedge \neg Y) \Rightarrow X$$

Solution: We use the following tableau derivations:

$$\begin{array}{c}
(X \wedge Y) \vee (X \wedge \neg Y) \Rightarrow X^{\mathsf{F}} \\
(X \wedge Y) \vee (X \wedge \neg Y)^{\mathsf{T}} \\
X^{\mathsf{F}} \\
\begin{array}{c|c}
X \wedge Y^{\mathsf{T}} & X \wedge \neg Y^{\mathsf{T}} \\
X^{\mathsf{T}} & X^{\mathsf{T}} \\
Y^{\mathsf{T}} & \neg Y^{\mathsf{T}} \\
\bot & \bot
\end{array}
\end{array}$$

Therefore there is no model. Thus the initial expression is unsatisfiable.

Therefore the expression

$$(X \wedge Y) \vee (X \wedge \neg Y) \Rightarrow X$$

is valid.

Problem 4.4 Find three models for the following proposition: 6pt

$$(P \vee Q \wedge R) \Rightarrow (P \vee Q) \wedge (\neg P \vee R)$$ 3min