Last Name: First Name:

Matriculation Number:

Exam
Artificial Intelligence 2

October, 2024

	To be used for grading, do not write here											
prob.	1.1	1.2	2.1	2.2	3.1	3.2	4.1	4.2	4.3	5.1	Sum	grade
total	8	10	9	10	10	9	9	12	7	7	91	
reached												

In the course Artificial Intelligence I/II we award bonus points for the first student who reports a factual error in an old exam. (Please report spelling/formatting errors as well.)

1 Probabilities

Problem 1.1 (Python)

1. Consider the Python program below. 4 pt

```
def foo(J,x):
    px = sum(J[x])
    res = []
    for y in range(len(J[x])):
      res.append(J[x][y]/px)
    return res
```

The input J represents the joint probability distribution of random variables X with domain $\{0, \dots, m-1\}$ and Y with domain $\{0, \dots, n-1\}$ in such a way that $J[x][y] = P(X = x, Y = y)$. The input x represents an element in the domain of X.
Which probability-related operation does the function `foo` compute?

Solution: The conditional probability distribution $P(Y = y \mid X = x)$.

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2. Assume a random variable X with domain $\{0, \dots, m-1\}$ and distribution D, i.e., $D[x] = P(X = x)$. 4 pt
Assume that $D[x]$ is non-zero for some even value of x.
Complete the definition of p in the program below in such a way that $p(D, x)$ returns the conditional probability $P(X = x \mid X \text{ is even})$, i.e., the probability of $X = x$ under the condition that X takes an even value.

```
def even(x):
  return x % 2 == 0

def p(D,x):
```

Solution:

```
def p(D,x):
  if even(x):
    sumEven = 0
    for i in range(len(D)):
        if even(i):
            sumEven += D[i]
    return D[x]/sumEven
  else:
    return 0
```

Problem 1.2 (Calculations)
Assume random variables X, Y both with domain $\{0, 1, 2\}$. For some outcomes A, the probabilities are known as follows:

A	$P(A)$
$X = 0 \wedge Y = 0$	a
$X = 0 \wedge Y \neq 0$	b
$X \neq 0 \wedge Y = 0$	c
$X \neq 0 \wedge Y \neq 0$	d

1. Give all subsets of the probabilities $\{a, b, c, d\}$ that must sum to 1. 2 pt

   Solution: Only $\{a, b, c, d\}$

2. In terms of a, b, c, d, give $P(X \neq 0)$ or argue why there is not enough information to compute the value. 2 pt

   Solution: $c + d$

3. In terms of a, b, c, d, give $P(X > Y)$ or argue why there is not enough information to compute the value. 2 pt

   Solution: Not enough information. The result is $c + P(X = 2, Y = 1)$, but we cannot compute the latter because there is no data that distinguishes between the values 1 and 2 for X or Y.

4. In terms of a, b, c, d, give $P(X = 0 \mid Y = 0)$ or argue why there is not enough information to compute the value. 2 pt

   Solution: $a/(a + c)$

5. Now assume we additionally know $e = P(X = Y)$. In terms of a, b, c, d, e, give $P(X + Y = 3)$. 2 pt

   Solution: $d - e + a$

2 Bayesian Reasoning

Problem 2.1 (Basic Rules)
Assume you are trying to relate economic development and your business results. You have collected the following data:

- The economy does well 40% of the time and badly otherwise.
- If your business does well, the economy does well 50% of the time.
- If the economy does well, your business does well 70% of the time.

You model the problem using two Boolean random variables E (economy does well) and B (business does well). You also abbreviate the events $E = true$ and $B = true$ as e and b.

1. By filling in the gaps below, state for each number in the text above, which probability it describes. 2 pt
   1. $P(\qquad\qquad) = 0.4$
   2. $P(\qquad\qquad) = 0.5$
   3. $P(\qquad\qquad) = 0.7$

   Solution:
   1. $P(e) = 0.4$
   2. $P(e \mid b) = 0.5$
   3. $P(b \mid e) = 0.7$

   ---

2. Calculate the probability that your business does well. 2 pt

   ---

   Solution: We have $P(e \mid b)P(b) = P(b \mid e) \cdot P(e)$. From that, we can calculate $P(b) = 0.7 \cdot 0.4/0.5 = 0.56$

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3. If your business does well, you estimate your earnings to be \$100/month, otherwise \$60/month. Calculate your expected utility (in \$/month) if the economy does well. 2 pt

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   Solution: $0.7 \cdot 100 + 0.3 \cdot 60 = 88$

   ---

4. Calculate the probability that your business and the economy are doing badly at the same time. 3 pt

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   Solution: We need $P(\neg b, \neg e)$. We know that
   - $P(b, e) = P(b \mid e) \cdot P(e) = 0.28$
   - $P(e) = P(b, e) + P(\neg b, e) = 0.4$, i.e., $P(\neg b, e) = 0.12$
   - $P(b) = P(b, e) + P(b, \neg e) = 0.56$ (from previous question), i.e. $P(b, \neg e) = 0.28$
   - $P(b, e) + P(\neg b, e) + P(b, \neg e) + P(\neg b, \neg e) = 1$, i.e., $P(\neg b, \neg e) = 0.32$

If the previous question was solved wrongly as w, the result should be $0.88 - w$.

Problem 2.2 (Bayesian Networks)
Consider the following situation about a day out.

- If the sun shines, you are out long, and you did not use sunscreen, you may get sunburned.
- If the sun shines or you meet friends, you may stay out long.

You want to model this situation as a Bayesian network using Boolean random variables S (sunshine), O (staying out long), B (sunburn), U (sunscreen used), and F (meeting friends).

1. Using a good variable ordering, model this as a Bayesian network. 2 pt

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Solution: Order: e.g., $FSOUB$. Network: edges from F, S to O, and from O, S, U to B.

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2. Assume you meet your friends and use sunscreen, and you want to determine the probability of sunburn. What are the query/evidence/hidden variables? 2 pt

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Solution: query: B, evidence: F, U, hidden: S, O

For the remaining questions, assume your network is $F \leftarrow S \rightarrow O \leftarrow B \rightarrow U$ (which may or may not be a correct solution to the previous question).

3. Which probabilities are stored in the conditional probability table of the node O? Which of those could be omitted and computed from the others? 3 pt

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Solution: $P(O = o \mid S = s, B = b)$ where o, b, f are Booleans, i.e., 8 entries in total. The entries for $O = false$ can be computed from the ones for $O = true$.

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4. Give the probability distribution $P(S \mid O = o, U = u)$ (for fixed o, u) in terms of the entries of the probability tables of the network. 3 pt

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Solution:

$$P(S \mid O = o, U = u) = P(S \mid O = o) = \alpha\big(P(S, O = o, B = true) + P(S, O = o, B = false)\big)$$

where for each Boolean s

$$P(S = s, O = o, B = b) = P(S = s) \cdot P(B = b) \cdot P(O = o \mid S = s, B = b)$$

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3 Markovian Reasoning

Problem 3.1 (Hidden Markov Models)
Consider the following situation:

- Each year the groundwater level at your location is high or not.
- Each year your harvests are good or not.
- You want to study the groundwater level by observing the harvests.

You choose to model this situation as a stationary first-order hidden Markov model with a stationary sensor model with Markov property, using two families of year-indexed Boolean random variables.

1. Give the state and evidence variables and their domains. 2 pt

Solution: State: G_a (groundwater level), evidence: H_a (harvests), all domains are the Booleans

2. Which probabilities do you need for this model? 3 pt

Solution: A transition model $T_{ij} = P(G_{a+1} = j \mid G_a = i)$ and a sensor model $S_{ij} = P(H_a = j \mid G_a = i)$. (Because all variables are Boolean, half of these entries are redundant.)

3. What would change about the answer to the previous question if the HMM were not stationary? 2 pt

Solution: We would need a different T for every year.

4. Assume a sequence $e_1, e_2, ...$ of Booleans such that e_a gives the quality of the harvest in year $a = 1, 2, ...$ Then the filtering algorithm can be written in recursive matrix form as 3 pt

$$\mathbf{f}_{1:a} = \alpha(O_a T^t \mathbf{f}_{1:a-1})$$

(where T^t is the transposed transition matrix).
Which probability distributions are represented by the $\mathbf{f}_{1:a}$, and how do we obtain the values of the O_a?

Solution: $\mathbf{f}_{1:a} = P(H_a \mid G_1 = e_1, \ldots, G_{n-1} = e_{n-1})$
O_a is the diagonal matrix whose diagonal contains the values of the distribution $P(H_a = e_a \mid X_a)$.

Problem 3.2 (Decision Processes and Utility)
Consider an agent moving along a rectangular grid of 3×3 locations.
The agent can stand still, or move 1 location up, down, left, or right except when already at the edge.
A movement step succeeds with probability 90%. In the remaining cases, the agent does not move.
The agent is initially in the bottom-left corner location. Its goal is to move to the top-right location.

1. Model this situation as a Markov Decision Process $\langle S, A, T, s_0, R\rangle$. Use a reward function that uses a constant reward for non-goal states. 5 pt

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Solution: One possible model is

- $S = \{0, 1, 2\} \times \{0, 1, 2\}$
- Let $M = \{(0,0), (0,1), (0,-1), (1,0), (-1,0)\}$. Then $A(s) = \{m \in M | s + m \in S\}$ where $+$ is component-wise addition.
- The transition model is given by
  - $a = (0,0)$: $T(s,a)(s) = 1$
  - otherwise: $T(s,a)(s+a) = 0.9$, $T(s,a)(s) = 0.1$
  - 0 for all other probabilities
- $s_0 = (0,0)$
- A typical choice is any function R that is high for the goal and slightly negative for other states. E.g., $R((2,2)) = 1$ and $R(s) = -0.1$ otherwise.

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2. Give an optimal policy π^*. 2 pt

Solution: Any policy is optimal that moves only up and right until at the goal and then stands still. E.g.: π^* maps $(2,2) \mapsto (0,0)$, $(i,j) \mapsto (1,0)$ if $i \neq 2$, and $(2,j) \mapsto (0,1)$ otherwise

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3. Now assume the agent is unable to tell whether an action resulted in movement. Explain (in about 2 sentences) how we can still represent this situation as an MDP. 2 pt

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Solution: We define a new MDP whose states are belief states, i.e., probability distribution over states $s \in S$. All actions are possible in every belief state, and cause transition between belief states. The reward of a belief state can be calculated as a probability-weighted of states.

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4 Learning

Problem 4.1 (Decision Trees and Lists)
Consider an unknown natural number $N \in \{1, \ldots, 10\}$. You are allowed to ask the following attributes/questions about N:

A Is N prime? (possibly answers: yes, no)

B What is N modulo 3? (possible answers: $0, 1, 2$)

C Is $N > 5$? (possibly answers: yes, no)

D Is N a root of $X^2 + X - 7$? (possibly answers: yes, no)

E What is the result of $\sin((N + 0.5)\pi)$? (possibly answers: $-1, 1$)

F What is N modulo 4? (possible answers: $0, 1, 2, 3$)

1. Give the shortest (in terms of tree depth) decision tree for identifying N that uses at most the above questions. 3 pt

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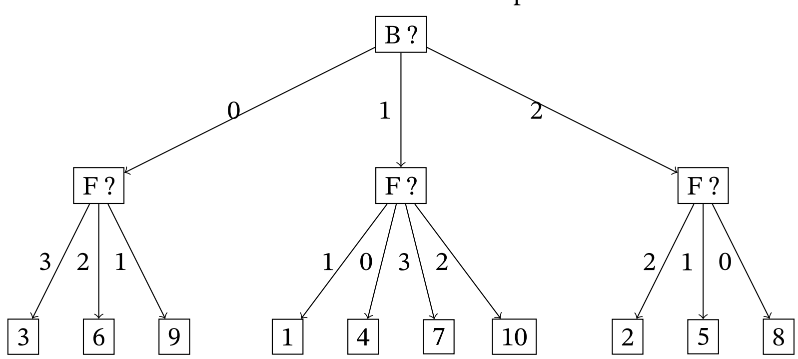
Solution: The tree must use B and F in either order. One possible tree is



Those have 3×4 possible answer combinations giving us a chance (but no guarantee) to disambiguate 10 options. Giving a concrete tree of depth 2 shows that we can indeed do it. No single question (and no other pair of questions) has ≥ 10 answer combinations, showing 2 is a lower bound.

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2. We can see each question as a random variable. Give the entropies $I(A)$, $I(C)$, and $I(D)$, i.e., the number of bits obtained by asking the question. (Simplify as much as possible without introducing approximate values.) 3 pt

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Solution: A: $I(\langle 2/5, 3/5 \rangle) = -2/5 \log_2(2/5) - 3/5 \log_2(3/5)$
C: 1
D: 0
Counting 1 as a prime number is wrong but was accepted anyway. In that case $I(A) = 1$.

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3. Now assume our goal is to learn the function that computes whether any natural number is a square number, given the answers to the questions A-F, using $N = 1$ and $N = 2$ for training data. Formally state this situation as an inductive learning problem $\langle \mathcal{H}, T \rangle$. 3 pt

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Solution: $\mathcal{H}$ is the set of functions $\mathbb{B} \times \{0, 1, 2\} \times \mathbb{B} \times \mathbb{B} \times \{-1, 1\} \times \{0, 1, 2, 3\} \rightarrow \mathbb{B}$ where $\mathbb{B} =$

$\{yes, no\}$. T is the set containing $(no, 1, no, no, -1, 1, yes)$ (for $N = 1$) and $(yes, 2, no, no, 1, 2, no)$ (for $N = 2$).
Alternatively, any subset of the hypothesis space can be used.

Problem 4.2 (Classifiers)
Consider a set E of examples of the form (x, y) where $x \in \mathbb{R}^2$ and $y \in \{0, 1\}$.
We want to learn a linear classifier h_w with a hard threshold. For the hard threshold, we use $\mathcal{T}(u)$ that returns 1 if $u > 0$ and 0 otherwise.

1. Give the general form of such a classifier. 2 pt

Solution: $h_w(x) = \mathcal{T}(w \cdot x)$ where $w \in \mathbb{R}^3$ and we embed x into $\mathbb{R}^3$ by putting $x_0 = 1$. Alternatively, we can write this as $h_w(x) = \mathcal{T}(w_1 \cdot x_1 + w_2 \cdot x_2 + w_0)$.

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2. Give a neural network that can be used to represent such a classifier. 3 pt

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Solution: One input cell for each x_1, x_2 and constant bias $x_0 = 1$. One perceptron output cell with input weight w_i for x_i.

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3. Give the formula for the squared error loss of such a classifier. 2 pt

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Solution: $\Sigma_{(x,y) \in E}(y - h_w(x))^2$

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4. Why is gradient descent not applicable to minimize the loss in this case? 1 pt

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Solution: $\mathcal{T}$ and thus the loss function are not differentiable.

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5. Instead, we can use the perceptron learning rule to update the weights. What is that rule, and how do we apply it using the examples from E? 2 pt

Solution: We start by putting $w_i = 0$ for all i. Then we apply for each example $(x, y) \in E$, the rule

$$w_i \leftarrow w_i + \alpha(y - h_w(x))x_i \quad \text{for each } i$$

where parameter α is the learning rate.

6. Now assume our classifiers are multi-layer neural networks and that gradient descent is applicable. Explain informally (in about 2 sentences) the basic idea of back-propagation. 2 pt

Solution: Applying gradient descent at the output layer yields updates for the connections from the last hidden layer. Aggregating these updates for each cell in the last hidden layer yields errors for the last hidden layer, which are used to recursively apply gradient descent for the preceding layer.

Problem 4.3 (Inductive Learning)
Consider the family tree given by the following relations:

couple	children
A, B	E, F
C, D	G
F, G	H,I

Assume we already know the predicate $\mathtt{par}(x, y)$ for x being a parent of y. Our goal is to learn the predicate $\mathrm{gp}(x, y)$ for x being a grandparent of y, i.e., to find a formula D such that $\forall x, y.\mathrm{gp}(x, y) \Leftrightarrow D(x, y)$.
We do not know D, but we have the following (counter-)examples for gp:

person-pair	grandparent
A, H	yes
B, I	yes
A, E	no
A, F	no
A, B	no
A, C	no

1. Give the intended formula D_1, i.e., the correct definition of grandparent. 2 pt

   Solution: $D_1(x, y) = \exists u.\mathtt{par}(x, u) \wedge \mathtt{par}(u, y)$

2. Give a formula D_2 that is true exactly for the positive examples. 2 pt

   Solution: $D_2(x, y) = (x = A \wedge y = H) \vee (x = B \wedge y = I)$

3. Explain (in about 3 sentences) the commonalities and pros and cons of learning the formula D as D_1 vs. D_2. 3 pt

   Solution: Both correctly learn the positive and the negative examples. D_2 is easy to learn, but it is of size $O(n)$ and overfits to the examples. D_1 is small and captures the intended formula, but is much harder to learn.

5 Natural Language Processing

Problem 5.1 (Information Retrieval)

1. Explain (in about 2 sentences) the connection between tfidf and cosine similarity to relate texts. 2 pt

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Solution: tfidf is a way to represent a text as a vector, relative to a corpus, by measuring the frequency of terms in some way. Cosine similarity uses the angle between two such vectors as a measure of how similar the texts represented by the vectors are.

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2. Explain (in about 2 sentences) the difference between information retrieval and information extraction. 2 pt

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Solution: Information retrieval identifies documents that are likely to contain information that answers the query. Information extraction obtains structured representations of parts of the content of a document, which can then be used to answer queries precisely.

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3. Explain (in about 3 sentences) what kind of function is learned in the continuous bag of words algorithm, as used e.g. in Word2Vec, and how the training proceeds. 3 pt

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Solution: It learns a word embedding, i.e., a function from words to vectors. Going through a corpus, each word is processed together with the context, i.e., the n words before and after. The neural network is trained to predict the word from its context.
