Last Name: First Name:

Matriculation Number:

Retake Exam
Artificial Intelligence 2

April 8, 2024

Please ignore the QR codes; do not write on them, they are for grading support

	To be used for grading, do not write here											
prob.	1.1	1.2	2.1	2.2	2.3	3.1	3.2	4.1	4.2	5.1	5.2	Sum	grade
total	7	8	7	3	10	12	10	10	8	7	8	90	
reached													

The “solutions” to the exam/assignment problems in this document are supplied to give students a starting point for answering questions. While we are striving for helpful “solutions”, they can be incomplete and can even contain errors even after our best efforts.
In any case, grading student’s answers is not a process of simply “comparing with the reference solution”, therefore errors in the “solutions” are not a problem in this case. If you find “solutions” you do not understand or you find incorrect, discuss this on the course forum and/or with your TA and/notify the instructors. We will – if needed – correct them ASAP.

In the course Artificial Intelligence I/II we award bonus points for the first student who reports a factual error in an old exam. (Please report spelling/formatting errors as well.)

1 Probabilities

Problem 1.1 (Python)

1. We use Python objects p to hold the joint probability distribution of random variables X and Y, i.e, $p[i][j] = P(X = i, Y = j)$. 4 Points
Consider the Python *program* below:

```
def bar(p):
    res = []
    for i in range(len(p)):
        s = 0
        q = p[i]
        for j in range(len(q)):
           s += q[j]
        res.append(s)
    return res
```

Which probability-related operation does `bar` implement?

Solution: The probability distribution of X, i.e., `bar(p)[i]` is $P(X = i)$.

2. Possibly using `bar` from above, write a function *cond* such that `cond(p,i,j)` returns the conditional probability $P(Y = j|X = i)$. 3 Points

```
def cond(p,i,j):
```

Solution:

```
def cond(p,i,j):
  return p[i][j]/bar(p)[i]
```

Problem 1.2 (Calculations)
Assume *random variables* X, Y both with *domain* $\{0, 1, 2\}$ with the following conditional probability distribution:

x	y	$P(X = x\|Y = y)$
0	0	a
0	1	b
0	2	c
1	0	d
1	1	e
1	2	f
2	0	g
2	1	h
2	2	i

1. Give all *subsets* of $\{a, b, c, d, e, f, g, h, i\}$ whose elements *sum* to 1. 2 Points

Solution: $\{a, d, g\}$, $\{b, e, h\}$, $\{c, f, i\}$

2. In terms of the values $a, b, c, d, e, f, g, h, i$, give $P(X \neq 0|Y = 0)$. 2 Points

Solution: $d + g$

3. Which of the following values can be computed from the values $a, b, c, d, e, f, g, h, i$? 2 Points
   - ☐ $P(X = 0)$
   - ☐ $P(Y = 0)$
   - ☐ $P(Y = 0|X = 0)$
   - ☐ $P(X = 0|Y = 0)$

Solution: Only the last one.

4. Which property of the values $a, b, c, d, e, f, g, h, i$ holds iff X and Y are independent? 2 Points

Solution: $a = b = c \wedge d = e = f \wedge g = h = i$

2 Bayesian Reasoning

Problem 2.1 (Bayesian Calculations)
Consider a disease with a prevalence of 1/1000, i.e., 1 in 1000 people have it. You are using a test that gives a yes/no answer for whether a person has the disease. However, the test randomly returns the wrong result 1% of the time.

1. Model this situation using random variables. State all probabilities whose values are given in the text. 3 Points

Solution: Boolean RVs D (for whether someone has the disease) and T (for the test result).
$P(D^+) = 1/1000$
$P(T^+|D^+) = 99/100$ (equivalently: $P(T^-|D^+) = 1/100$)
$P(T^-|D^-) = 99/100$ (equivalently: $P(T^+|D^-) = 1/100$).

2. Calculate the probability of a test returning yes. 2 Points

Solution: Marginalization: $P(T^+) = P(T^+|D^+) \cdot P(D^+) + P(T^+|D^-) \cdot P(D^-) = 0.00099 + 0.00999 = 0.01098$.

3. You are using the test on a person, and it returns yes. Calculate the *probability* that she has the disease. 2 Points

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Solution: Bayes rule: $P(D^+|T^+) = P(T^+|D^+) \cdot P(D^+)/P(T^+) = 99/1098 = 11/122$

Problem 2.2 (Conditional Bayes Rule)

1. Consider 3 Boolean random variables X, Y, C. We write x, y, c for the events where the corresponding variable is true. Prove that 3 Points

$$P(x|y,c) = P(y|x,c) \cdot P(x|c)/P(y|c)$$

Solution: This follows immediately from

$$P(x|y,c) \cdot P(y|c) \cdot P(c) = P(x,y,c) = P(y|x,c) \cdot P(x|c) \cdot P(c)$$

after canceling $P(c)$.

Problem 2.3 (Bayesian Networks)
Consider the following situation about a car:

- Your car is unusable if it is out of gas or if it is broken. These two are the only causes.
- You might be late for work if your car does not work or if you oversleep. These two are the only causes.

You want to model this situation as a *Bayesian network* using Boolean random variables.

1. Give an appropriate set of random variables and their meaning. Give a good *variable ordering* and draw the resulting *Bayesian network*. 3 Points

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Solution: Variables: C (car unusable), G (out of gas), B (broken), L (late for work), S (overslept).
Order: e.g., G, B, C, S, L
Network: $G \to C \leftarrow B$ and $C \to L \leftarrow S$

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2. Give the probability of the car being unusable in terms of the entries of the conditional probability table of your network. 2 Points

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Solution: $P(C^+) = \Sigma_{b,g\in\{true,false\}} P(C^+|B=b, G=g) \cdot P(B=b) \cdot P(G=g)$

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3. Now you decide to make the car-unusable node deterministic. Explain (in about 2 sentences) why that choice is justified based on the description above, and how it affects the conditional probability table of that node. 2 Points

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Solution: The description says that the car **is** (rather than e.g., "might be") unusable if is a broken or without gas, i.e., that the relation is deterministic and not governed by probability. Formally: $P(C^+|G^+ \vee B^+) = 1$. Thus, we do not have to store a CPT for C and only need to store the function $C = G||B$.

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4. Now you decide to make the late-for-work node a noisy disjunction node. Explain (in about 2 sentences) which two properties must hold about its probability distribution for this decision to be justified. Judge if these are backed by the description. 3 Points

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Solution:

Firstly, the two causes must be the only causes, i.e., $P(L^+|C^-, S^-) = 0$. This is explicitly stated in the description.
Secondly, the two causal relationships must be independent of each other. Formally, if both causes are present, the probability of non-lateness must be the product of the two inhibition factors: $P(L^+|C^+, S^+) = 1 - P(L^-|C^+, S^-) \cdot P(L^-|C^-, S^+)$. This is not commented on by the description. Common sense background knowledge indicates that the probably of being late is even higher if both causes are present, e.g., if oversleeping prevents catching a bus.

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3 Markovian Reasoning

Problem 3.1 (Hidden Markov Models)
Consider the following situation, which you want to model as a hidden Markov model:

- You make daily observations about your business (B), which can go well (b_1), average (b_2), or badly (b_3).
- This is caused by the weather (W), which can be good (w_1) or bad (w_2). Over a period of days d, you have collected the following probabilities for this causal relationship:

$$S_{ij}^d = P(B_d = b_j | W_d = w_i) = \begin{pmatrix} 0.4 & 0.4 & 0.2 \\ 0.5 - 1/(2d) & 0.5 & 1/(2d) \end{pmatrix}$$

- The weather is influenced by the previous day's weather as follows:
  - If the weather is good, it stays good 60% of the time.
  - If the weather is bad, it stays bad 30% of the time.

1. Give the *state* and *evidence variables* and their *domains*. 2 Points

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Solution: evidence variables $B_d \in \{b_1, b_2, b_3\}$, *state variables* $W_d \in \{w_1, w_2\}$.

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2. Fill in the following sentence: 2 Points
The transition matrix is given by

$$T_{ij} = P(\qquad\qquad\qquad) = \begin{pmatrix} \qquad\qquad\qquad \end{pmatrix}$$

Solution:

$$T_{ij} = P(W_{d+1} = w_j | W_d = w_i) = \begin{pmatrix} 0.6 & 0.4 \\ 0.7 & 0.3 \end{pmatrix}$$

3. Explain (in about 2 sentences) whether the Markov model and the sensor model are stationary. 2 Points

Solution: The Markov model is stationary because T does not depend on d. The sensor model is not stationary because S^d depends on d.

4. There is a 75% chance for the weather to be good at day $d = 1$. Calculate the resulting probability distribution for the business at day 1. 2 Points

Solution: $P(B_1) = \langle 0.75, 0.25 \rangle \cdot S^1 = \langle 0.3 + 0, 0.3 + 0.125, 0.15 + 0.125 \rangle = \langle 0.3, 0.425, 0.275 \rangle$.

This problem is continued on the next page

Now assume we have made observations e_d of B_d for a sequence of days.
Consider the filtering algorithm in matrix form:

$$f_{1:d+1} = \alpha \cdot O_{d+1} \cdot T^t \cdot f_{1:d}$$

where α is a normalization factor.

5. State the definition of O_d in terms of e_d. 2 Points

Solution: If $e_d = w_i$, then O_d is the diagonal matrix formed from the i-th column of S^d.

6. If $f_{1:1}$ is the distribution of W_1, which distribution is given by $f_{1:d}$? 2 Points

Solution: $P(W_d | B_1 = e_1, \dots, B_d = e_d)$

Problem 3.2 (Decision Processes and Utility)
Consider an *agent* moving along 8 locations as indicated below.

$$\begin{array}{ccccccc} 0 & \leftrightarrow & 1 & \leftrightarrow & 2 & \leftrightarrow & 3 \\ \updownarrow & & \downarrow & & \uparrow & & \updownarrow \\ 7 & \leftrightarrow & 6 & \leftrightarrow & 5 & \leftrightarrow & 4 \end{array}$$

The *agent*'s movement is as follows:

- In every state, it can make moves called -1, 0, and 1.
- Additionally, in states 1 and 5, it can make a move called 5.
- Each move n, made in location l moves the agent to location $l \oplus n$ (where $\oplus$ is addition modulo 8). However, 10% of the time, a move fails, and the agent's location does not change.

The *agent*'s *goal* is to move to location 7.

1. Model this situation as a *Markov Decision Process*. 4 Points

Solution: One possible model is

- $S = \{0, \dots, 7\}$
- $A(s) = \{-1, 0, 1\} \cup \{5 | s = 5\}$
- The transition model is given by
  - $P(s \mid s, 0) = 1$
  - for $a \neq 0$: $P(s \oplus a \mid a, s) = 0.9$ and $P(s \mid a, s) = 0.1$.

  All other probabilities are 0.
- A typical choice is any *function* R that is high for the *goal* and slightly negative for other *states*. E.g., $R(7) = 1$ and $R(s) = -0.1$ otherwise.

From now on, consider the policy π defined by $\pi(7) = 0$ and $\pi(s) = 1$ otherwise.

2. Calculate the probability distribution of the agent's location after starting in location 0 and making 2 moves according to π. 2 Points

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Solution: $P(s_0) = \langle 1, 0, 0, 0, 0, 0, 0, 0 \rangle$
$P(s_1) = \langle 0.1, 0.9, 0, 0, 0, 0, 0, 0 \rangle$
$P(s_2) = \langle 0.01, 0.18, 0.81, 0, 0, 0, 0, 0 \rangle$
where the vectors contains the probabilities for the values $0, \dots, 7$.

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3. State the equation for evaluating the policy π, which we can use to iteratively calculate the utility of each state under policy π. Include the initial values of the utilities. 2 Points

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Solution: $U(s) = R(s) + \gamma \Sigma_{s' \in S} U(s') \cdot P(s' \mid s, \pi(s))$
Initially $U(s) = 0$.

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4. Assume we have already calculated the utility $U(s)$ of each state s. Explain (in about 2 sentences, including the relevant formulas) how can we determine if π is optimal? 2 Points

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Solution: For each s, we check if $\pi(s)$ maximizes the expected utility $EU(s, a)$. That is given by $\Sigma_{s' \in S} P(s' | s, a) \cdot U(s')$.

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4 Learning

Problem 4.1 (Decision Trees and Lists)
Consider a *word* W chosen uniformly from $\{bad, bed, bend, pend, pad, ped\}$. You are allowed to ask the following questions about W:

A length of the word
B first letter of the word
C second letter of the word
D last letter of the word

1. Show that there is no decision tree for W of depth 2. 2 Points

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Solution: All questions have at most 2 possible answers, so a decision tree of depth 2 has at most 4 leaves. But we need at least 5 leaves to cover all options for W.

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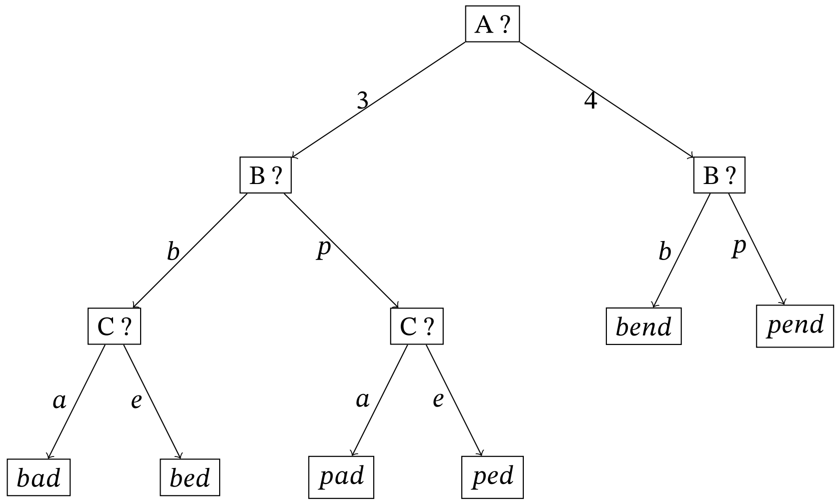
2. Draw the *decision tree* for W that arises from asking the questions in the order A,B,C,D. (Do not ask additional questions if the word can already be identified.) 3 Points

Solution: The tree is



3. Calculate the information gain for all 4 questions. 2 Points

Solution: A, C: $-1/3\log_2 1/3 - 2/3\log_2 2/3$
B: $-1/2\log_2 1/2 - 1/2\log_2 1/2 = 1$
D: $-1\log_2 1 = 0$

4. Which question would the information gain algorithm ask first? 1 Points

Solution: B

5. Give two words such that removing them from the choices for W makes the *determination* $\{A, B\} \succ W$ hold. 2 Points

Solution: Any pair out of $\{bad, bed\} \times \{pad, ped\}$

Problem 4.2 (Statistical Learning)
You observe the values below for 50 games of a tennis player. You want to predict the result based on time of day and opponent.

		Number of	
Time	Opponent	wins	losses
Morning	Weaker	5	1
Afternoon	Weaker	6	2
Evening	Weaker	3	0
Morning	Similar	3	3
Afternoon	Similar	2	3
Evening	Similar	4	5
Morning	Stronger	2	2
Afternoon	Stronger	1	3
Evening	Stronger	1	4

1. What is the hypothesis space for this situation, seen as a decision tree learning problem? 2 Points

Solution: The set of functions

$$\{Morning, Afternoon, Evening\} \times \{Weaker, Similar, Stronger\} \to \{Win, Loss\}$$

2. Explain (in about 2 sentences) the key characteristic of this data that makes decision tree learning inapplicable, and under what circumstances it would be applicable. 2 Points

Solution: The results are not uniquely determined by the input. So no decision tree can exist. If each row had a 0 in one of the Win/Loss columns, it would be applicable.

3. Now instead, consider this as a statistical learning problem. As hypotheses, we use the probability distributions $P(Result|Time, Opponent)$.
Relative to the observed data, give the likelihood of the following hypothesis: The player's probability to win is 80% if the game is against a weaker player, and it is 10% otherwise. 2 Points

Solution: The likelihood is the probability of the data under the condition that the hypothesis holds. This is $0.8^{14} \cdot 0.2^3 \cdot 0.1^{13} \cdot 0.9^{20}$

4. To learn a hypothesis via Bayesian learning, we model this situation as a Bayesian network $Time \to Result \leftarrow Opponent$. Give the resulting entries of the conditional probability table for 2 Points
    1. $P(Opponent = Weaker) =$
    2. $P(Result = win \mid Time = Afternoon, Opponent = Weaker) =$

Solution: $P(Opponent = Weaker) = 0.34$
$P(Result = win \mid Time = Afternoon, Opponent = Weaker) = 0.75$

5 Natural Language Processing

Problem 5.1 (Grammars)

Consider the following *probabilistic grammar*:

$$\begin{array}{lcl} S & \to & NP\ VP[1] \\ NP & \to & Article\ Noun[0.6] \mid Name[0.4] \\ VP & \to & Verb[0.5] \mid TransVerb\ NP[0.5] \\ Article & \to & \text{the}[0.7] \mid \text{a}[0.3] \\ Noun & \to & \text{stench}[0.2] \mid \text{breeze}[0.3] \mid \text{wumpus}[0.5] \\ Name & \to & \text{John}[0.3] \mid \text{Mary}[0.7] \\ Verb & \to & \text{smells}[1] \\ TransVerb & \to & \text{sees}[0.6] \mid \text{shoots}[0.4] \end{array}$$

1. Which of the above productions comprise the lexicon? 1 Points

Solution: $Noun$ to $TransVerb$

2. Explain (in about 2 sentences) why it is practical to separate the lexicon from the other productions. 2 Points

Solution: The production in the lexicon are usually much more numerous and much less standardized. Moreover, they only occur as leaves of the tree and are thus not essential for the grammatical structure.

3. Give the *probability* of the *sentence* 2 Points

   Mary shoots the breeze.

Solution: $0.4 \cdot 0.7 \cdot 0.5 \cdot 0.4 \cdot 0.6 \cdot 0.7 \cdot 0.3 = 0.007056$

4. Explain (in about 2 sentences) how we can use a treebank to learn the probabilities of the productions. 2 Points

Solution: For every production p for non-terminal L, we count how often it occurs in a subtree in the treebank, say n_L. Then we count how many of those subtrees use the production p, say n_p. We learn the probability n_p/n_L.

Problem 5.2 (Information Retrieval)
Consider the corpus $D = \{d_1, d_2, d_3\}$ where

- d_1: "The man is tall."
- d_2: "The tall man sees the woman."
- d_3: "The woman shouts at the tall man."

Below we use alphabetical order for the vector components:

at, is, man, sees, shouts, tall, the, woman

1. Give the vector $tf(\_, d_3)$ 2 Points

Solution: $tf(\_, d_3) = \langle 1/7, 0, 1/7, 0, 1/7, 1/7, 2/7, 1/7 \rangle$.

2. Give the vector $idf(\_, D)$. 2 Points

Solution: $idf(\_, D) = \log_{10}(3/\langle 1,1,3,1,1,3,3,2\rangle) = \langle k,k,0,k,k,0,0,l\rangle$ with $k = \log_{10} 3$ and $l = \log_{10} 1.5$.

3. State the definition of $tfidf$. 2 Points

Solution: $tfidf(t,d,D) = tf(t,d) \cdot idf(t,D)$

4. Explain (in about 2 sentences) the point of using the inverse document frequency in the definition of $tfidf$. Use the word the and the corpus D as an example. 2 Points

Solution: $idf(t,D)$ can be used as a measure of the relevance of a word for characterizing a document — words with low idf occur in many documents and are thus less distinctive.
For example, $idf(the, D) = 0$ and thus occurrences of the word the are ignored when calculating the $tfidf$ vectors.