Name:

# Retake Exam <br> Künstliche Intelligenz 2 

Feb. 19., 2018

## You have 90 min (sharp) for the test;

Write the solutions to the sheet.
The estimated time for solving this exam is 67 minutes, leaving you 23 minutes for revising your exam.

You can reach 134 points if you solve all problems. You will only need 125 points for a perfect score, i.e. 9 points are bonus points.

Different problems test different skills and knowledge, so do not get stuck on one problem.

|  | To be used for grading, do not write here |  |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| prob. | 1.1 | 1.2 | 2.1 | 2.2 | 2.3 | 3.1 | 3.2 | 3.3 | 4.1 | 4.2 | 4.3 | 4.4 | Sum | grade |
| total | 14 | 16 | 8 | 8 | 20 | 8 | 6 | 12 | 8 | 6 | 8 | 20 | 134 |  |
| reached |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Please consider the following rules; otherwise you may lose points:

- If you continue an answer on another page, please indicate the problem number on the new page and give a page reference on the old page.
- Always justify your statements (we would like to give poins for incorrect answers). Unless you are explicitly allowed to, do not just answer "yes", "no", or " 42 ".
- If you write program code, give comments!


## 1 Bayesian Reasoning

## Problem 1.1 (AFT Tests)

Trisomy 21 (Down syndrome) is a genetic anomaly that can be diagnosed during pregnancy 14 pt using an amniotic fluid test.

The probability of a fetus having Down syndrome is strongly correllated with the age of the mother during pregnancy. For 25 year old mothers the probability is $0.08 \%$, for 43 year old mothers it increases to $2 \%$ (we only consider those two age groups).

However, diagnostic tests are never perfect. We distinguish two kinds of errors:

- Type I Error (False Positive): The test result is positive even though the child is healthy.
- Type II Error (False Negative): The test result is negative even though the child has trisomy 21.

The probabilities of Type I and Type II Errors are both merely 1\% for amniotic fluid tests for Down syndrome.

1. Express all the numbers given above as conditional probabilities. Use the random variable $F$ with Domain $\left\{A g e_{25}, A g e_{43}\right\}$ for the age of a mother and the Boolean random variables Pos and Down for the propositions "The amniotic fluid test is positive" and "The child has Down syndrome" respectively.
2. Assume now we have a fixed mother of age 25 (i.e. for any event $X$ you may assume $\left.P\left(X \mid F=A g e_{25}\right)=P(X)\right)$. How can we compute the probability that a child has Down syndrome, given that the amniotic fluid test is positive? Give an equation that only uses the probabilities for which we have actual numbers.

## Problem 1.2 (Bayesian Networks)

Consider the following Bayesian network with boolean variables:


1. Which nodes in the network are

- Stochastically independent
- Conditionally independent and under which conditions?

2. What exactly (formal criterion) does an arrow between two nodes in a bayesian network mean for the associated events?

## 2 Decision Theory

Problem 2.1 (Decision Preferences)

1. Name and state three of the axioms for preferences (i.e. $\succ$ ).
2. How are preferences related to value functions?

Problem 2.2 (Expected Utility)
What is the formal(!) definition of expected utility? Explain every variable in the defining 8 pt equation.

## Problem 2.3 (Decision Network)

You need a new car. Your local dealership has two models on offer - $C_{1}$ for $1500 \$$ and 20 pt $C_{2}$ for $1250 \$$. Either car can be of good quality or bad quality. If $C_{1}$ is of bad quality, repairing it will cost $200 \$$, if $C_{2}$ is of bad quality repairing it will cost $500 \$$. You have the choice between two tests:

1. You can either take $C_{1}$ on a short test drive for 30 minutes, which will confirm that it is of good quality with certainty $75 \%$ probability, and that it is of bad quality (if it is) with certainty $65 \%$.
2. Or you can borrow $C_{2}$ for a whole weekend, which will confirm that it is of good quality with $80 \%$ certainty, and that it is of bad quality (if it is) with certainty $70 \%$.

The a priori probability that $C_{1}$ is of good quality is $70 \%$ and for $C_{2}$ it is $65 \%$.

1. Draw the decision network for which test to apply and which car to buy in either case. (assume a meaningful utility function given).
2. Explain formally how to compute which test to apply.
3. Assume you apply test 1 and it seems like the car is of bad quality. Explain how to compute which car you should actually buy.

## 3 Markov Models

Problem 3.1 (Bellman Equation)
State the Bellman Equation and explain 1) every symbol in the equation and 2) what the 8 pt equation is used for and how.

## Problem 3.2 (Stationary)

Define what it means for a Markov model to be stationary, and why we are interested in 6 pt stationarity.

## Problem 3.3 (Stock Market Predictions)

You bought SpaceY stock recently and try to predict whether to buy more or sell. The 12 pt stock market is in one of two possible states; bull state or bear state. In a bull state, it will (in the long term) be advantageous to buy stock; in a bear state it will be more advantageous to sell.

If the market is in a bull state, the probability it will still be in a bull state tomorrow is $60 \%$. If it is in a bear state, the probability it will remain so tomorrow is $80 \%$.

If the market is in a bull state, the probability that your stock will rise that day is $90 \%$. If it is in a bear state, your stock will more likely fall (with $60 \%$ probability).

1. Explain what kind of probabilities prediction, filtering and smoothing compute in this scenario.
2. Give the underlying equations for the first two of these algorithms and explain what each variable in the equation represents.

## 4 Learning

Problem 4.1 (Gradient Descent)
Explain a gradient descent algorithm.

Problem 4.2 (Information Entropy)
Explain and define information entropy.
6 pt

## Problem 4.3 (Overfitting)

Explain what overfitting means and why we want to avoid it.
8 pt

## Problem 4.4 (Tennis Trees)

Consider the following decisions on whether or not to go play tennis. The target is 20 pt "PlayTennis".

| Outlook | Temperature | Humidity | Wind | PlayTennis |
| :--- | :--- | :--- | :--- | :--- |
| Sunny | Hot | High | Weak | No |
| Overcast | Hot | High | Weak | Yes |
| Rain | Mild | High | Weak | Yes |
| Rain | Cool | Normal | Strong | No |
| Rain | Mild | Normal | Weak | Yes |

Explain how to apply decision tree learning to this table. In particular, define the notion of information entropy.

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