Assignment8 – Learning

Given: June 20   Due: June 25

Problem 8.1 (Support Vectors)

Consider the following 2-dimensional dataset

support vector	*classification*
$\mathbf{x}_1 = \langle 0, 0\rangle$	$\mathbf{y}_1 = -1$
$\mathbf{x}_2 = \langle 0, 0.5\rangle$	$\mathbf{y}_2 = -1$
$\mathbf{x}_3 = \langle 0.5, 0\rangle$	$\mathbf{y}_3 = -1$
$\mathbf{x}_4 = \langle 1, 1\rangle$	$\mathbf{y}_4 = 1$
$\mathbf{x}_5 = \langle 2, 2\rangle$	$\mathbf{y}_5 = -1$

1. Give a *linear separator* in the form $h(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$ for the dataset containing only the examples for $\mathbf{x}_1$ to $\mathbf{x}_4$.
2. Explain informally why no linear separator exists for the full dataset of all 5 vectors.
3. Transform the dataset into a 3-dimensional dataset by applying the function $F(\langle u, v\rangle) = \langle u^2, v^2, u + v\rangle$.
4. Give a *linear separator* for the transformed full dataset in the form $h(\mathbf{x}) = \mathbf{w} \cdot \mathbf{x} + b$.

Problem 8.2 (Weight Updates)

We're trying to find a linear separator. Our examples are the set

Example number	$\mathbf{x}_1$	$\mathbf{x}_2$	y
1	2	0	2
2	3	1	2

Our hypothesis space contains the functions $h_{\mathbf{w}}(\mathbf{x}) = A(\mathbf{w} \cdot \mathbf{x})$ for 2+1-dimensional vectors $\mathbf{w}, \mathbf{x}$ (using the trick $\mathbf{x}_0 = 1$ to allow for the constant term $\mathbf{w}_0$) and some fixed function A.

As the initial weights, we use $\mathbf{w}_0 = \mathbf{w}_1 = \mathbf{w}_2 = 0$.

For each of the following cases, iterate the respective weight update rule once for each example (using the examples in the order listed). Use learning rate $\alpha = 1$.

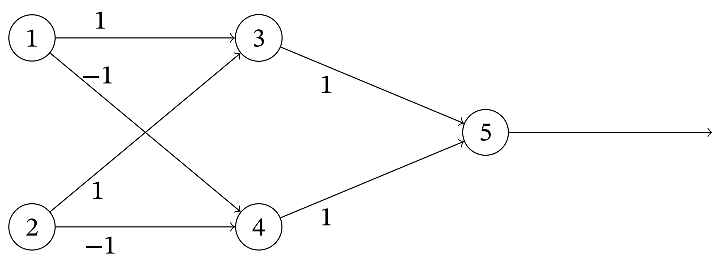
1. Using the threshold function $A(z) = \mathcal{T}(z)$, i.e., $A(z) = 1$ if $z > 0$ and $A(z) = 0$ otherwise. Here we cannot do gradient descent, so we have to use the perceptron learning rule.
2. Using the logistic function $A(z) = 1/(1 + e^{-x})$. Here we use gradient descent.

Problem 8.3 (XOR Neural Network)

Consider the following neural network with

- inputs a_1 and a_2
- units 3, 4, 5 with activation functions such that $a_i \leftarrow \begin{cases} 1 & \text{if } \Sigma_j w_{ji} a_j > b_i \\ 0 & \text{otherwise} \end{cases}$

- weights w_{ij} as given by the labels on the edges



1. Assume $b_3 = b_4 = b_5 = 0$ and inputs $a_1 = a_2 = 1$. What are the resulting activations a_3, a_4, and a_5?
2. Choose appropriate values for b_3, b_4, and b_5 such that the network *implements* the XOR function.