Assignment3 – Decisions and Markov Models

Given: May 16   Due: May 23

Problem 3.1 (Decision Network)

You try to decide on whether to take an umbrella (boolean variable M) to Uni. Obviously, it is useful (numeric variable U for the utility) to do so if it rains (boolean variable W) when you go back home, but it is annoying to carry around if it does not even rain. You decide based on whether the weather forecast predicts rain (boolean variable F).

1. Draw the decision network for bringing/leaving an umbrella depending on the weather forecast and the actual weather. Explain the four variables M, U, W, and F and what to store in their probability tables.
2. Explain formally how to compute whether or not to take an umbrella, assuming you know the probability that the forecast is correct.

Problem 3.2 (The Value of Information)

Chef Giordana runs a kitchen that provides food for a large organisation. A salad is sold for €6 and costs €4 to prepare. So a sold salad is profit of €2, and an unsold salad a loss of €4. Actual demand will be 40 (with probability 0.5) or 60 (also with probability 0.5) each day.

Each day, Giordana must decide in advance between two options: prepare 40 or 60 salads.

1. In the absence of additional information, compute the expected utility of each decision option, and choose the best option.
2. She is considering a new ordering system, where she knows the demand perfectly in advance. So she can always choose the better of the two options. State the formula for computing the value of this perfect information, explain the components in Giordana's case, and compute the value.

Problem 3.3 (Markov Mood Detection)

On any given day d, your roommate Moody is either happy or sad, so $M_d \in \{h,s\}$. Usually when he is sad, he stays sad for a while, and $P(M_{d+1} = s \mid M_d = s) = 0.7$. But aside from that he is a cheery guy, and $P(M_{d+1} = h \mid M_d = h) = 0.85$.

He either listens to jazz or metal music, so $L_d \in \{j,m\}$. On a happy day he usually listens to Jazz, and $P(L_d = j \mid M_d = h) = 0.7$. On a sad day, he slightly prefers metal, and $P(L_d = m \mid M_d = s) = 0.6$.

1. Model this situation as a Markov process. Explain what the state and evidence variables are. What order does the process have? Is the transition model stationary? Is the sensor model stationary? Does the transition model have the Markov property? Does the sensor model have the sensor Markov property? Explain all answers in one short sentence each.
2. State the formula for the full joint probability distribution. You know that he was happy at day d_0. What is the probability that he is happy and plays Jazz for the next two days?

Problem 3.4 (Moody HMM)

Consider the Markov process from before about the roommate Moody (which in particular gives the concrete probabilities needed below). We have already modeled it as an HMM with state variables M_d and evidence variables L_d.

Because the transition model is first-order and stationary, we can collect the conditional probabilities for the state transitions into a matrix $T_{ij} = P(M_d = x_j \mid M_{d-1} = x_i)$ where x_i, x_j are two states (i.e., two possible values of the state variable). We use S for the number of states, and T is an $S \times S$ matrix.

Because the sensor model is stationary and has the sensor Markov property, we can collect the conditional probabilities for the observations into a matrix $O_{ij} = P(L_d = y_i \mid M_d = x_j)$ where x_j is a state and the y_i are the possible observations. If there are N possible observations, this is an $N \times S$ matrix. For a fixed observation e, the diagonal $S \times S$ matrices O_e from the lecture notes are obtained from the rows of this matrix.

1. Clarify the modeling as an HMM. Concretely:
   2. What is S? Give the transition matrix T.
   3. What is N? Give the sensor matrix O.

   Note that you need to choose and state orderings for the states and observations so that it is clear which state/observation corresponds to which row/-column of T and O.
4. Now consider a fixed sequence $L_1 = e_1, L_2 = e_2$ of observations that we have made for two days. Concretely, you heard Moody play metal on day $d = 1$ and jazz on day $d = 2$.
   5. Give the diagonal sensor matrices O_1 and O_2 corresponding to the observation at $d = 1$ and $d = 2$.
   6. You are not sure what kind of mood your flatmate was in on day $d = 0$, but it was either good or bad with equal probability. The HMM algorithm for filtering and smoothing uses compact matrix/vector equation to compute f and b. Use those equation to determine the probability distribution of Moody's mood on day $d = 1$.
      Note that T^t in the filtering equation in the lecture notes denotes the transpose of T.
      Grading will focus on writing out the matrices with the correct probabilities in them and on formally stating the computations that need to applied to those matrices. You should also actually do those computations, but that is secondary.