This exercise sheet gets you started with Prolog. To complete the assignments you can use

- a Prolog interpreter such as downloadable from http://www.swi-prolog.org/
- an online interpreter such as available at https://swish.swi-prolog.org.

However, for submission you will have to upload a single Prolog file for each problem. The files must run using the SWI Prolog interpreter. The manual for SWI Prolog is available at http://www.swi-prolog.org/pldoc/.

Now and for all future programming problems, your solutions must

- strictly follow the specification in the problem statement, in particular regarding the names and arities of the predicates — we may do automated testing!
- include comments that explain how your code works including example invocations of your programs — a key grading criterion is how easy it is for the grading tutor to verify that you solved the problem correctly.

Problem 1.1 (Basic Prolog Functions) 35 pt

Implement the functions listed below in Prolog. Note that many of them are built-in, but we ask you create your own functions.

1. a function reversing a list
   Test case:
   ```prolog
   ?- myReverse([1,2,3,4,2,5],R).
   R = [5, 2, 4, 3, 2, 1].
   ```

2. a function removing multiple occurrences of elements in a list
   Test case:
   ```prolog
   ?- removeDuplicates([1,1,1,1,2,2,3,4,1,2,7],A).
   A = [1, 2, 3, 4, 7].
   ```
   **Hint:** You may want to implement a helper method `delete(X,LS,RS)` that removes all instances of `X` in `LS` and returns the result in `RS`.

3. a function for zipping two lists
   zip takes two lists and outputs a list of pairs (represented as 2-element lists) of elements at the same index in the two lists. If the lists do not have the same length, the zipped list contains only as many pairs as the shorter list.
   Create a Prolog predicate with 3 arguments: the first two are the two lists to zip and the third one the result. For instance:
   ```prolog
   ?- zip([1,2,3],[4,5,6],L).
   L = [[1, 4], [2, 5], [3, 6]].
   ?- zip([1,2],[3,4,5],L).
   L = [[1, 3], [2, 4]].
   ```

4. a function for computing permutations of a list
   Try it out on paper first and understand why this is difficult.
   Test case:
Note that there are two ways for specifying such a function:

(a) return a list of all permutations
(b) return a single permutation each time such that Prolog finds them one by one.

Here we are using the second way, i.e., myPermutations(L,P) must in particular be true if P is some permutation of L.

Hint: One possible solution is to start with a helper predicate takeout(X,L,M) that is true iff M is the result of removing the first occurrence of X from L. Or equivalently: M arises by adding X somewhere in L. How does this allow you to define the notion of permutation recursively?

Solution:

1. the reverse function

   % myReverseAcc uses an additional argument (the second one) as an accumulator
   % in which the result is built.
   % Its invariant is that myReverserAcc(X,Y,Z) iff reverse(X);Y = Z.
   % When the first argument is empty, we return the accumulated result.

   myReverserAcc([],X,X).
   myReverserAcc([X|Y],Z,W) :- myReverserAcc(Y,[X|Z],W).

   To compute the reversal, we initialize the accumulator with the empty list.

   myReverse(A,R) :- myReverserAcc(A,[],R).

2. the remove duplicates function

   delete(_,[],[]).
   delete(X,[X|T],R) :- delete(X,T,R).
   delete(X,[H|T],[H|R]) :- not(X=H), delete(X,T,R).

   removeDuplicates([],[]).
   removeDuplicates([H|T],[H|R]) :- delete(H,T,S), removeDuplicates(S,R).

3. the zip function

   zip([],[]).
   zip([L],[L]).
   zip([H1|T1],[H2|T2],[[H1,H2]|T]) :- zip(T1,T2,T).

4. the permute function

   takeout(X,[X|T],T).
   takeout(X,[H|T1],[H|T2]) :- not(X=H), takeout(X,T1,T2).

   % There is exactly one permutation of the empty list.
   myPermutations([],[]).
   % To find a permutation P of a longer list [H|T], we permute T into Q
   % and insert H somewhere into Q.
   myPermutations([H|T],P) :- myPermutations(T,Q), takeout(P,H,Q).

Note that we defined takeout in such a way that the second argument is input and the third one output. But Prolog does not distinguish input and output: when we use it later, we use the third argument as input and the second one as output.
Problem 1.2 (Binary Tree)

A binary tree of (in this case) natural numbers is inductively defined as either

- an expression of the form \( \text{tree}(n, t_1, t_2) \) where \( n \) is a natural number (the label of the node) and \( t_1 \) and \( t_2 \) are themselves binary trees (the children of that node)

- or nil for the empty tree. (Normally a tree cannot be empty, but it is more convenient here to allow an empty tree as well.)

In particular, the nodes of the form \( \text{tree}(n, \text{nil}, \text{nil}) \) are the leaf nodes of the tree, the others are the inner nodes.

An example tree in Prolog would be:

\[
\text{tree}(1, \text{tree}(2, \text{nil}, \text{nil}), \text{tree}(2, \text{nil}, \text{nil}))
\]

1. Write a Prolog function \texttt{construct} that constructs a binary tree out of a list of (distinct) numbers such that for every subtree \( \text{tree}(n, t_1, t_2) \) all values in \( t_1 \) are smaller than \( n \) and all values in \( t_2 \) are larger than \( n \).

Note that there are usually multiple such trees for every list. One example is:

\[
\text{?- construct([3,2,4,1,5], T).}
\]

\[
T = \text{tree}(3, \text{tree}(2, \text{tree}(1, \text{nil}, \text{nil}), \text{nil}), \text{tree}(4, \text{nil}, \text{tree}(5, \text{nil}, \text{nil}))).
\]

2. Write Prolog functions \texttt{count_nodes} and \texttt{count_leaves} that take a binary tree and return the number of nodes and leaves, respectively.

3. Write a Prolog function \texttt{symmetric} that checks whether a binary tree is symmetric.

Solution:

% add(X,S,T) inserts a node with label X into tree S yielding tree T
% Inserting into the empty tree yields a tree with a single node.
add(X,nil,tree(X,nil,nil)).

% To insert an element smaller than the root, insert on the left.
add(X,tree(Root,L,R),tree(Root,L1,R)) :- X @< Root, add(X,L,L1).

% To insert an element bigger than the root, insert on the right.
add(X,tree(Root,L,R),tree(Root,L,R1)) :- X @> Root, add(X,R,R1).

% To construct a binary tree T, from a list L, we insert all elements in order.
% We use an accumulator that we initialize with the empty tree.
construct(L,T) :- constructAcc(L,T,nil).
% At the end of the list, we return the accumulator.
constructAcc([],T,T).
% For each element of the list, we add it to the accumulator A (obtaining A1) and recurse.
constructAcc([N|Ns],T,A) :- add(N,A,A1), constructAcc(Ns,T,A1).

% The empty tree has no nodes.
count_nodes(nil,0).
% An inner node has one more node than its child trees together.
count_nodes(tree(_,L,R),N) :- count_nodes(L,NL), count_nodes(R,NR), N is NL+NR+1.
% Note that we do not need an additional case for leaf nodes here.

% The empty tree has no leaves.
count_leaves(nil,0).
% A leaf node has 1 leaf (itself).
count_leaves(tree(_,nil,nil),1).
% An inner node has as many leaves as its child trees together.
count_leaves(tree(_,L,R),N) :- count_leaves(L,NL), count_leaves(R,NR), N is NL+NR.

% The empty tree is symmetric.
symmetric(nil).
% Any other tree is symmetric if its two child trees are mirror images of each other.
symmetric(tree(_,L,R)) :- mirror(L,R).

% The empty tree is its own mirror image.
mirror(nil,nil).
% Otherwise, the mirror image arises by mirroring and swapping the child trees.
mirror(tree(X,L1,R1),tree(X,L2,R2)) :- mirror(L1,R2), mirror(R1,L2).

%A few tests
test1(X) :- construct([5,2,4,1,3],Y), count_leaves(Y,X).
% X=2
test2(X) :- construct([6,10,5,2,9,4,8,1,3,7],Y), count_leaves(Y,X).
% X=3
symmetric(tree(1,tree(2,nil,nil),tree(2,nil,nil))).
% true.
symmetric(tree(1,tree(3,nil,nil),tree(2,nil,nil))).
% false.

Problem 1.3
For each of the following agents, develop a PEAS description of the task environment.

1. Robot soccer player
2. Internet book-shop agent (that is: an agent for book shops that stocks up on books depending on demand)
3. Autonomous Mars rover
4. Mathematical theorem prover
5. First-person shooter (Counterstrike, Unreal Tournament etc.)

Additionally, characterize the environments of these agents according to the properties discussed in the lecture. Where not “obvious”, justify your choice with a short sentence.
Finally, choose suitable designs for the agents.