Assignment12 – ALC

Problem 12.1 (ALC)

Consider the following description logic signature

- *concept* symbols: i (for instructor), s (for student), c (for *course*), p (for program)
- *role* symbol m (for is-member-of) used for
  - *instructors* giving a *course*
  - *students* taking a *course*
  - *students* being enrolled in a *degree program*
  - *courses* being part of a *degree program*

We use an extension of $\mathcal{ALC}$, in which there are dual roles: there is a role m^{-1} that captures the relation has-as-member, e.g., $MK\ m\ AI$ iff $AI\ m^{-1}\ MK$.

1. For the *signature* above, give a *concept axiom* that captures that instructors can only be members of *courses*.

   Solution: $i \sqsubseteq \forall m.c$

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2. Give a *concept axiom* for the above *signature* that captures: *courses* that are taken by a *student*, must be given by an *instructor*.

   Solution: $c \sqcap \exists m^{-1}.s \sqsubseteq \exists m^{-1}.i$

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3. Calculate the translation to *first-order logic* of $s \sqsubseteq \forall m.\exists m.p$.

   Solution: $\forall x.s(x) \Rightarrow (\forall y.m(x,y) \Rightarrow (\exists z.m(y,z) \wedge p(z)))$

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4. Given a *first-order model* $\langle \mathcal{D}, \mathcal{I} \rangle$, define an appropriate case of the *interpretation* mapping for the formula $\forall r^{-1}.C$.

   Solution: $[\![\forall r^{-1}.C]\!] = \{u \in D |$ for $v \in \mathcal{D},$ if $(v,u) \in [\![r]\!]$, then $v \in [\![C]\!]\}$

Problem 12.2 (ALC Semantics)

Consider the $\mathcal{ALC}$ *concepts* $\forall R.(C \sqcap D)$ and $\forall R.C \sqcap \forall R.D$.

1. By applying the semantics of $\mathcal{ALC}$, show that the two are equivalent.

Solution: We have:

$$[\![\forall R.(C \sqcap D)]\!]$$
$$= \{x \in \mathcal{D} | \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in [\![C \sqcap D]\!]\}$$
$$= \{x \in \mathcal{D} | \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in ([\![C]\!] \cap [\![D]\!])\}$$

$$[\![\forall R.C \sqcap \forall R.D]\!]$$
$$= [\![\forall R.C]\!] \cap [\![\forall R.D]\!]$$
$$= \{x \in \mathcal{D} | \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in ([\![C]\!] \cap Q)\}$$

where

$$Q = \{x \in \mathcal{D} | \text{ for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in [\![D]\!]\}$$

Now to prove that sets are equal, consider an $x \in \mathcal{D}$ and see that both conditions are equivalent to

$$\text{for all } y \in \mathcal{D}, \text{ if } (x,y) \in [\![R]\!], \text{ then } y \in [\![C]\!] \text{ and } y \in [\![D]\!]$$

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2. Translate both formulas to first-order logic and state which FOL formula we would need to prove (e.g., with the ND calculus) to show that the two are equivalent.

Solution: The translation yields

$$C_1(x) = \forall y.R(x,y) \Rightarrow C(y) \wedge D(y)$$
$$C_2(x) = (\forall y.R(x,y) \Rightarrow C(y)) \wedge (\forall y.R(x,y) \Rightarrow D(y))$$

We need to show

$$\forall x.C_1(x) \Leftrightarrow C_2(x)$$

Problem 12.3 (ALC TBox)

Consider $\mathcal{ALC}$ with the following

- primitive concepts: woman, man
- roles: has_child, has_parent, has_sibling, has_spouse

Give an $\mathcal{ALC}$ *TBox* that defines the *concepts* person, parent, mother, father, grandmother, aunt, uncle, sister, brother, onlychild, cousin, nephew, niece, fatherinlaw, motherinlaw.

Solution: $\mathsf{person} = \mathsf{man} \sqcup \mathsf{woman}$

$\mathsf{parent} = \mathsf{person}\exists\mathsf{has\_child.person}$
$\mathsf{mother} = \mathsf{womanparent}$
$\mathsf{father} = \mathsf{manparent}$
$\mathsf{grandmother} = \mathsf{woman}\exists\mathsf{has\_child.parent}$
$\mathsf{aunt} = \mathsf{woman}\exists\mathsf{has\_sibling.parent}$
$\mathsf{uncle} = \mathsf{man}\exists\mathsf{has\_sibling.parent}$
$\mathsf{sister} = \mathsf{woman}\exists\mathsf{has\_sibling.person}$
$\mathsf{brother} = \mathsf{man}\exists\mathsf{has\_sibling.person}$
$\mathsf{onlychild} = \mathsf{person}\overline{\mathsf{brother} \sqcup \mathsf{sister}}$
$\mathsf{cousin} = \mathsf{person}\exists\mathsf{has\_parent.}\exists\mathsf{has\_sibling.parent}$
$\mathsf{nephew} = \mathsf{man}\exists\mathsf{has\_parent.}\exists\mathsf{has\_sibling.person}$
$\mathsf{niece} = \mathsf{woman}\exists\mathsf{has\_parent.}\exists\mathsf{has\_sibling.person}$
$\mathsf{fatherinlaw} = \mathsf{man}\exists\mathsf{has\_child.}\exists\mathsf{has\_spouse.person}$
$\mathsf{motherinlaw} = \mathsf{woman}\exists\mathsf{has\_child.}\exists\mathsf{has\_spouse.person}$
