Assignment11 – FOL-Inference, Knowledge Representation

Problem 11.1 (First-Order Resolution)

Let $P, Q \in \Sigma_1^p$, $R \in \Sigma_2^p$, $c, d \in \Sigma_0^f$. Prove the following *formula* using the *first-order resolution calculus* $\mathcal{R}_1$.

$$\exists X.\forall Y.\exists W.\exists Z.\neg((R(Z,Y)\lor\neg P(Z))\land(\neg Q(d)\lor P(c))\land(Q(d)\lor\neg P(c))\land(\neg R(Z,Y)\lor\neg P(W)\lor\neg Q(X))\land P(c))$$

Hint: Note, that the formula is already (close to) a negated CNF, so if you spend any significant amount of time transforming the formula, you are most likely doing something wrong.

Solution: We negate:

$$\forall X.\exists Y.\forall W.\forall Z.(R(Z,Y)\lor\neg P(Z))\land(\neg Q(d)\lor P(c))\land(Q(d)\lor\neg P(c))\land(\neg R(Z,Y)\lor\neg P(W)\lor\neg Q(X))\land P(c)$$

Substituting bound variables:

$$(R(Z,f_Y(X))\lor\neg P(Z)\lor\neg Q(d)\lor P(c)\lor Q(d)\lor\neg P(c)\lor\neg R(Z,f_Y(X))\lor\neg P(W)\lor\neg Q(X)\lor P(c))$$

Resolution:

$$\begin{aligned}
\{Q(d)^{\mathsf{T}}, P(C)^{\mathsf{F}}\} + \{P(c)^{\mathsf{T}}\} &\Longrightarrow \{Q(d)^{\mathsf{T}}\}\\
\{Q(d)^{\mathsf{T}}\} + \{R(Z,f_Y(X))^{\mathsf{F}}, P(W)^{\mathsf{F}}, Q(X)^{\mathsf{F}}\}[d/X] &\Longrightarrow \{R(Z,f_Y(d))^{\mathsf{F}}, P(W)^{\mathsf{F}}\}\\
\{P(c)^{\mathsf{T}}\} + \{R(Z,f_Y(d))^{\mathsf{F}}, P(W)^{\mathsf{F}}\}[c/W] &\Longrightarrow \{R(Z,f_Y(d))^{\mathsf{F}}\}\\
\{P(c)^{\mathsf{T}}\} + \{R(Z,f_Y(X))^{\mathsf{T}}, P(Z)^{\mathsf{F}}\}[c/Z] &\Longrightarrow \{R(c,f_Y(X))^{\mathsf{T}}\}\\
\{R(c,f_Y(X))^{\mathsf{T}}\}[d/X] + \{R(Z,f_Y(d))^{\mathsf{F}}\}[c/Z] &\Longrightarrow \emptyset
\end{aligned}$$

Problem 11.2 (First-Order Tableau)

Prove or refute the following formula using the first-order *free variable tableaux calculus*. We have $P, R \in \Sigma_1^p$ and $f \in \Sigma_1^f$.

$$(\forall X.P(X) \Rightarrow R(f(X))) \Rightarrow ((\exists X.P(X)) \Rightarrow (\exists.R(Y)))$$

Solution:

$$
\begin{array}{c}
(((\forall X.P(X) \Rightarrow R(f(X))) \Rightarrow (\exists.P(X))) \Rightarrow (\exists.R(Y)))^{\mathsf{F}} \\
(\forall X.P(X) \Rightarrow R(f(X)))^{\mathsf{T}} \\
(\exists X.P(X) \Rightarrow (\exists Y.R(Y)))^{\mathsf{F}} \\
(\exists X.P(X))^{\mathsf{T}} \\
(\exists Y.R(Y))^{\mathsf{F}} \\
(P(Z) \Rightarrow R(f(Z)))^{\mathsf{T}} \\
P(s)^{\mathsf{T}} \\
R(W)^{\mathsf{F}} \\
\begin{array}{c|c}
P(Z)^{\mathsf{F}} & R(f(s))^{\mathsf{T}} \\
\bot : [s/Z] & \bot : [f(s)/W]
\end{array}
\end{array}
$$

Problem 11.3 (Modeling in Description Logic)

Consider the following situation:

- Some beings are persons, some are animals.
- Persons and animals may like other persons or animals.
- Alice is a person, and she likes the animal Bubbles.

1. Model this situation as a semantic network. Explain the different kinds of nodes and edges occurring in your network.

   Solution: Kinds of nodes:

   - represent concepts: being, person, animal
   - represent individuals: Alice, Bubbles

   Kinds of edges:

   - assert that one concept is a subconcept of another (is-a): person-being, animal-being
   - assert that one individual is an instance of a concept (inst): Alice-person, Bubbles-animal
   - represent a relation between two concepts: an edge labeled 'like' relation person-animal (These edges can often be omitted.)
   - assert that two individuals are in a relation: an edge labeled 'like' Alice-Bubbles

2. Model the same situation in first-order logic and compare the results.

   Solution: Nodes (by kind):

- For each concept-node c, we need a unary predicate symbol: $\mathtt{being} \in \Sigma_1^p$, $\mathtt{person} \in \Sigma_1^p$, $\mathtt{animal} \in \Sigma_1^p$.
- For each individual-node i, we need a nullary function symbol: $\mathtt{Alice} \in \Sigma_0^f$, $\mathtt{Bubbles} \in \Sigma_0^f$.

Edges (by kind):

- For each subconcept-edge (is-a) from c to d, we need an axiom $\forall x.c(x) \Rightarrow d(x)$. Here: $\forall x.\mathtt{person}(x) \Rightarrow \mathtt{being}(x)$ and $\forall x.\mathtt{animal}(x) \Rightarrow \mathtt{being}(x)$.
- For each instance-edge (inst) from i to c, we need an axiom $c(i)$. Here: $\mathtt{person}(\mathtt{Alice})$ and $\mathtt{person}(\mathtt{Bubbles})$.
- For each relation-representing-edge r from c to d, we need a binary predicate symbol. Here: $like \in \Sigma_2^p$. To represent the subject and object concepts, we can add axioms $\forall x, y.r(x, y) \Rightarrow c(x)$ and $\forall x, y.r(x, y) \Rightarrow d(y)$. Here: $\forall x, y.\mathtt{like}(x, y) \Rightarrow \mathtt{person}(x)$ and $\forall x, y.\mathtt{like}(x, y) \Rightarrow \mathtt{animal}(y)$.
- For each relation-asserting-edge for r from i to j, we need axiom $r(i, j)$. Here: $\mathtt{like}(\mathtt{Alice}, \mathtt{Bubbles})$.

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3. Explain the difference between inst and is-a edges.

Solution: inst edges are between an individual and a concept. They correspond to $\in$ math.
is-a edges are between two concepts. They correspond to $\subseteq$ in math.

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4. Explain the difference between having a relation edge between two concepts vs. asserting a relation between two individuals.

Solution: The former models that a relation r exists between individuals of the respective concepts. That corresponds to a FOL-predicate symbol in Σ_2^p. For each pair of individuals the relation may be true or false.
The latter asserts that that relation holds for two individuals. For example, an r-edge from i to j corresponds to the FOL-axiom $r(i, j)$.
