Assignment10 – Inference in First-Order Logic

Problem 10.1 (Unification)
Let $S_1 \in \Sigma_2^f$, $S_2 \in \Sigma_3^f$, $f \in \Sigma_1^f$, $g \in \Sigma_2^f$, $c \in \Sigma_0^f$
Decide whether (and how or why not) the following pairs of terms are unifiable.

1. $S_1(g(f(x), g(x, y)), y)$ and $S_1(g(z, v), f(w))$
2. $S_2(g(f(x), g(x, u)), f(y), z)$ and $S_2(g(g(g(u, v), f(w)), f(c)), f(g(u, v)), f(c))$

Problem 10.2 (Natural Deduction)
Let $R \in \Sigma_2^p$, $P \in \Sigma_1^p$, $c \in \Sigma_0^f$.
Prove the following formula in Natural Deduction:

$$(\forall X.\forall Y.R(Y, X) \Rightarrow P(Y)) \wedge (\exists Y.R(c, Y)) \Rightarrow P(c)$$

Problem 10.3 (First-Order Tableaux)
Prove or refute the following formula using the first-order *free variable tableaux calculus*. We have $P, Q \in \Sigma_1^p$.

$$(\forall X.P(X) \Rightarrow Q(X)) \Rightarrow (((\forall X.P(X)) \Rightarrow (\forall.Q(X))))$$