Assignment2 – Bayesian Networks

Given: May 8   Due: May 13

Problem 2.1 (Probabilities in Python)
Complete the partial *implementation* of probabilities at `https://kwarc.info/teaching/AI/resources/AI2/probabilities/`

Problem 2.2 (Bayesian Rules)
Give the formulas and a one-sentence explanation of the following basic rules in Bayesian inference:
1. Bayes rule
2. Product rule
3. Chain rule
4. Marginalization
5. Normalization

Problem 2.3 (Is your TA in the office?)
You want to discuss something with your TA. You know that

1. the probability of your TA being in the office, assuming it is morning, is $\frac{1}{5}$,

2. if your TA is in the office, there is a $\frac{1}{3}$ probability it is morning,

3. the probabilities that it is morning or afternoon are both $\frac{1}{2}$

Your tasks:
1. Write down the probabilities mentioned above as formulas
2. Compute the full joint probability distribution
3. What's the probability you'll meet your TA, if you come to the office in the afternoon?

Problem 2.4 (AFT Tests)
Trisomy 21 (*Down syndrome*) is a genetic anomaly that can be diagnosed during pregnancy using an amniotic fluid test.
The probability of a foetus having Down syndrome is strongly correlated with the age of the pregnant parent. We will only consider the following two age groups.

1. For 25 year olds the probability is one in 1250,

2. for 43 year old parents it increases to one in fifty.

However, diagnostic tests are never perfect. We distinguish two kinds of errors:

3. Type I Error (False Positive): The test result is positive even though the child is healthy.

4. Type II Error (False Negative): The test result is negative even though the child has trisomy 21.

The probabilities of Type I and Type II Errors are both merely 1% for amniotic fluid tests for Down syndrome.

1. Express the four items above in the form of conditional probabilities. Use the random variable F with domain $\{Age_{25}, Age_{43}\}$ for the age of the pregnant person and the Boolean random variables Pos and $Down$ for the propositions "*The amniotic fluid test is positive*" and "*The child has Down syndrome*" respectively.

2. Assume that we have a 25 year old pregnant person. Using Bayes' theorem, express and compute the probability that their child has Down syndrome, given that the amniotic fluid test is positive. What can we conclude from the result?

Problem 2.5 (Medical Bayesian Network)
Both Malaria and Meningitis can cause a fever, which can be measured by checking for a high body temperature. Of course you may also have a high body temperature for other reasons. We consider the following random variables for a given patient:

- Mal: The patient has malaria.

- Men: The patient has meningitis.

- HBT: The patient has a high body temperature.

- Fev: The patient has a fever.

Consider the following Bayesian network for this situation:

$$HBT$$
$$\uparrow$$
$$Fev$$
$$\nearrow \quad \nwarrow$$
$$Mal \qquad Men$$
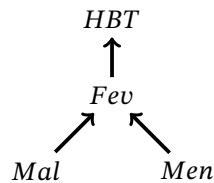
1. Explain the purpose of the edges in the network regarding the conditional probability table.
2. What would have happened if we had constructed the network using the variable order Mal, Men, HBT, Fev? Would that have l better network?
3. How do we compute the probability distribution for the patient having malaria, given that he has high body temperature? State the query variables, hidden variables and evidence and write down the equation for the probability we are interested in.