Assignment8 – Calculi for Propositional Logic

Problem 8.1 (FOL-Signatures)

1. Model the following situation as a FOL signature. (FOL and PLNQ signatures are the same.)
   - We have constants (= nullary functions) called `zero` and `one`.
   - We have a binary function called `plus`.
   - We have a unary function called `minus`.
   - We have a binary predicate called `less`.
2. Now consider the signature given by
   - $\Sigma_0^f = \{a, b\}$
   - $\Sigma_1^f = \{f, g\}$
   - $\Sigma_2^f = \{h\}$
   - $\Sigma_0^p = \{p\}$
   - $\Sigma_1^p = \{q\}$
   - $\Sigma_2^p = \{r\}$
   - all other sets empty
3. Give a term over this signature that uses all function symbols
4. Give a formula over this signature that uses all function and predicate symbols

Problem 8.2 (Natural Deduction)

Prove the following formula using the propositional Natural Deduction calculus.

$$(A \vee B) \wedge (A \Rightarrow C) \wedge (B \Rightarrow C) \Rightarrow C$$

Problem 8.3 (Proving in Tableau Calculus)

We use the *propositional variables* P, Q, and R and define *formulae* A, B, and C by

$$A = Q \wedge (Q \Rightarrow R)$$
$$B = P \Rightarrow A$$
$$C = P \Rightarrow R$$

Prove the *formula* $B \Rightarrow C$ using the *propositional tableau calculus* $\mathcal{T}_0$.

Problem 8.4 (Logical Systems)

Fix a set V of propositional variables. We define a logical system $\langle L, K, \models \rangle$. (Note: This logical system is different from the ones in the lecture and only used here as an exercise.)

- L is the powerset of V, i.e., a formula is a set of propositional variables.
- K is the set of functions $V \to \{F, T\}$.
- For $A \in L$ and $M \in K$, $M \models A$ holds if $M(p) = T$ for all $p \in A$.

1. Give examples of formulas that are
    1. satisfiable
    2. falsifiable
    3. unsatisfiable
    4. valid

    Give a sound and complete calculus for this logical system.
2. Consider the relation $H \vdash A$ holding if $A = \bigcup_{h \in H} h$. Check if $\vdash$ is a derivation relation.