MathWebSearch 0.5: Scaling an Open Formula Search Engine

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Abstract. MathWebSearch is an open-source, open-format, content-oriented search engine for mathematical formulae. It is a complete system capable of crawling, indexing, and querying expressions based on their functional structure (operator tree) rather than their presentation.
In version 0.5, we concentrate on scalability issues in MathWebSearch to take advantage of corpora in the giga-formula range. We re-implemented the index to make it distributable and made all the APIs web standards conformant. Our experiments show that this architecture results in a scalable application.

1 Introduction

As the world of information technology grows, being able to quickly search data of interest becomes one of the most important tasks in any kind of environment, be it academic or not. Here we tackle the problem of finding information that is given in the form of (mathematical) formulae. Standard search engines like Google cannot deal with formulae at all, severely limiting the reach and utilization of technical, scientific, and engineering documents.

In this paper we present new work in the context of the MathWebSearch system; a search engine that addresses the problem of searching mathematical formulae from a semantic point of view, it finds formulae by their structure and meaning not via their presentation.

In [KŞ06] we have presented the motivation, query language, and web front end of MathWebSearch 0.1. In [KK07] we have re-examined the value proposition of semantic search for mathematical knowledge homing in on the benefits and sacrifices induced by the various search approaches [You06b; MM06; LM06], from a user's perspective. The result of this analysis is MathWebSearch 0.5, which we describe in this paper. The new version features significant efficiency gains (space efficiency increased by a factor of five), new management features, advanced searching capabilities, and a new user interface. The MathWebSearch system (see [MWS] for details) is released under the Gnu General Public License.

The motivation for the work reported in this paper is the availability of large[1] corpora, such as the arXMLiv corpus [SK08] with almost three quarters of a million scientific articles and an estimated giga-formula. This has not

[1] We deem a corpus as *large* if it has more than 20 million expressions

only re-kindled interest in formula search[2], but also severely taxes the scalability of systems. Scalability issues for presentation-based search engines have been addressed in [SL11]. Such engines map formulae to "special words" which can then be indexed by conventional bag-of-word information retrieval engines, which have become extremely scalable over the last years. The case for MathWebSearch is completely different, since the content-based unification queries it offers require an index data structure that reflects the inner structure of formulae (rather than just pointers to words). Even with the space efficiency gains in MathWebSearch 0.5, the indices will surpass the main memory of most machines. Therefore, we have laid the foundations for distributing the MathWebSearch in this version.

Before we present MathWebSearch 0.5 from a technical perspective in Section 3, we will recap unification-based querying. We evaluate the system on a large corpus in Section 4 and see that we need to distribute MathWebSearch to cope with linear RAM usage. Section 5 presents the necessary extensions of the indexing. Section 6 concludes the paper and discusses future work.

2 Querying Mathematics by Unification

Retrieval of mathematical knowledge and information via unification-based queries for content-encoded mathematical formulae is very natural. In [KŞ06] we have already discussed **instantiation queries**, which can be used to retrieve partially remembered formulae, e.g. the query for the formula for energy of a given signal $s(t)$ in Figure 1. Note that instantiation queries are more expressive as a query language than e.g. regular expressions supported by some text-based search engine, since we can use variable co-occurrences to query for co-occurring subterms.

Query (query variables marked as named boxes)	Result (Parseval's Theorem)
$\int_{\boxed{min}}^{\boxed{max}} \boxed{f}(x)^2 dx$	$\frac{1}{T}\int_0^T s^2(t)dt = \sum_{k=-\infty}^{\infty} \|c_k\|^2$

Fig. 1. An Instantiation Query

To see the full power of unification-based querying consider a student who encounters $\int_{\mathbb{R}^2} |\sin(t)\cos(t)|dt$ and wishes to know if there are any mathematical statements (like theorems, identities, inequalities) that can be applied to it. Indeed, there are many such statements (for example Hölder's inequality) and they can be found using **generalization queries**. The idea behind answering generalization queries is that the index marks universal[3] variables in subterms as generalization targets. Hence, the search engine looks for terms in the index

[2] The next NTCIR-10 Challenge in spring 2013 will have a "math track". NTCIR evaluates information access technologies in a series of competition events in Japan

[3] We consider an identifier as universal if it can be instantiated without changing the truth value of the containing expression. In formal representations like first-order

which, after instantiating the universal identifiers, become equal to the query. For our example, we have in the index the term (we reuse the box notation for generalization targets in the index) in Figure 2, which the search engine instantiates $\boxed{x} \mapsto t, \boxed{f} \mapsto sin, \boxed{g} \mapsto cos, \boxed{D} \mapsto \mathbb{R}^2$ in order to find the generalization query. Note that the variant query $\int_{\mathbb{R}^2} |\sin(t)\cos(2t)|dt$ will not find Hölder's inequality since that would introduce inconsistent substitutions $\boxed{x} \mapsto t$ and $\boxed{x} \mapsto 2t$.

$$\int_{\boxed{D}} \left|\boxed{f}(\boxed{x})\boxed{g}(\boxed{x})\right| d\boxed{x} \leq \left(\int_{\boxed{D}} \left|\boxed{f}(\boxed{x})\right|^{\boxed{p}} d\boxed{x}\right)^{\frac{1}{\boxed{p}}} \left(\int_{\boxed{D}} \left|\boxed{g}(\boxed{x})\right|^{\boxed{q}} d\boxed{x}\right)^{\frac{1}{\boxed{q}}}$$

Fig. 2. A Formula with Universal Variables in the Index

A very similar idea is used in **variation queries** where the indexed terms are searched to match the search expression but only renamings of generic terms are allowed. This type of queries prove to be helpful when the structure of the term needs to be maintained.

Sometimes, however, one is in the position that the searching criteria is somewhere between instantiation queries (i.e. parts are unknown) and generalization queries (parts are probably instantiated already). In this case we give the possibility to pose **unification queries**. As the name suggests, the query just finds terms which are unifiable with the search expression. A query like $g^2\cos(\boxed{x}) + b\sin(\sqrt{y})$ would match the term $\boxed{a}\cos(\boxed{t}) + \boxed{b}\sin(\boxed{t})$ as we can substitute $\boxed{x} \mapsto \sqrt{y}, \boxed{t} \mapsto \sqrt{y}, \boxed{a} \mapsto g^2, \boxed{b} \mapsto b$ to get the term $g^2\cos(\sqrt{y}) + b\sin(\sqrt{y})$.

3 The MathWebSearch System, Version 0.5

The MathWebSearch system consists of the three main components pictured in Figure 3. The *crawler subsystem* collects data from the corpora[4]. It transforms the mathematical formulae in the corpus into *MWS Harvest*s (XML files that contain formula-URIreference pairs) and feeds them into the core system. The *core system* (the MathWebSearch daemon mwsd) builds the search index and processes search queries: it accepts the MathWebSearch input formats (*MWS Harvest* and *MWS Query*; see [KP]) and generates the MathWebSearch output format (*MWS Answer Set*). These are communicated through the *RESTful interface* restd which provides a public HTTP API conforming to the REST paradigm.

logic, such variable occurrences can be effectively computed, but in semi-formal settings like mathematical textbooks, they have to be approximated by heuristic methods; see the discussion in the conclusion for details.

[4] Note that we envision essentially one crawler per corpus. The crawlers are specialized to the respective formula representation, the organization and access methods to the corpus, etc. We have only implemented a crawler for the arXiv (see Section 4), but additional crawlers can be patterned after this (see Section 6.1).

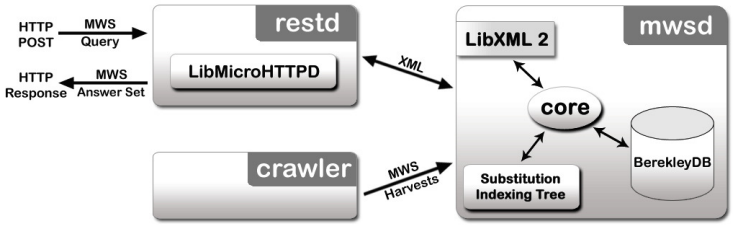


Fig. 3. MWS-0.5 System Structure

These components have been implemented using POSIX-compliant [Pos] C^{++}. We use the MicroHTTPd library [Mic] API for handling HTTP, and LibXML2 [Vei] API for XML parsing. The meta-data accompanying the internal index is stored using an external database system. As we are dealing mainly with key-value retrieval, the BerkeleyDB [Ber] API was preferred.

The system supports two main workflows:

1. The crawler sends an *MWS Harvest* to mwsd. The XML is parsed and an internal representation is generated. This is used to update the Substitution Indexing Tree and consequently the database.
2. The user sends an *MWS Query* to mwsd. The XML is parsed, an internal query is generated. Using an efficient traversal of the index tree, formulas matching the search term are retrieved and aggregated into a result. This is translated to an *MWS Answer Set* and sent back to the user.

3.1 Substitution Tree Indexing in MathWebSearch

As we are interested in indexing mathematical formulae at a large scale (document archives, text corpora), repetitive content is expected. After all, theorems are built on top of other theorems and terms on top of subterms. With this in mind, we chose a space-efficient internal representation based on substitutions.

In the previous version of MathWebSearch, we used a technique borrowed from Automated Theorem Proving called Substitution Indexing [Gra96]. It involves indexing expressions in a tree based on generality. The root is a generic variable and, as we go from a node to one of its children, one or more substitutions occur. For this version, we kept the substitution tree model and performed a few changes to better fit our design goals.

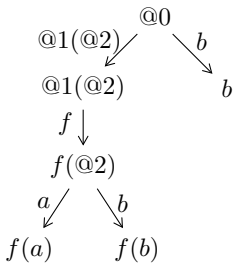


Fig. 4. Example DFS Substitution Tree

As such, we improved query times, by imposing a fixed substitution ordering. Hence, the query term describes a deterministic path through the index. The chosen ordering instantiates the left-most variable first, equivalent to DFS traversal of the operator tree. An example in-

dex, containing the terms $f(a)$, $f(b)$ and b, is presented in Figure 4. Note that the edges in the tree are labeled with the operators and operations: @1(@2) stands for the application operation.

Additionally, we save space by performing two steps of pre-processing for inserted terms, as well as query terms.

Firstly, we detect and reduce identical subterms. This is done by breaking the term into all possible subterms and detect equivalent subterms. Following this, the term is rewritten using only unique subterms (repeated matches are replaced through references to the first match in DFS order). For example the term $f(a, g(b), g(b))$ can be rewritten to $f(a, g(b), @[4])$, where $@[4]$ represents the 4^{th} term in DFS order: the subterms in DFS order are: $f(a, g(b), @[4])$, f, a, $g(b)$, g, b.

Furthermore, query terms with repeating identical query variables are reduced and handled as query terms with no repeating query variables. More importantly, this makes the search process stateless, as no previously matched query variable instantiations need to be stored.

Secondly, we hold an internal dictionary which maps symbols (in Content-MathML, represented by element name, attributes and text content) to integer IDs. The encoding relies on the fact that there are relatively few distinct tokens (compared to the number of expressions, for example). This achieves significant memory savings at a small price, since each (inserted or query) term is encoded exactly once.

3.2 Search Front ends and Embeddings

For practical applications, mwsd serves as a search back-end that needs to be embedded into a front-end system, which hides some of the complexities of writing MathWebSearch queries from the user. One example of a front-end system we are experimenting with is given in Figure 5[5] Here the user can enter queries in the LaTeX extended with the `\qvar` macro for query variables. This is then transformed into the content-MathML-based MathWebSearch queries by the LaTeXML daemon [GSK11] (the formula is also presented to the user with the query variables colored red). Upon receiving the resulting URIs, the frontend assembles a list of formulae and paper titles which link to the original paper. In this situation we are making use of the fact that TeX/LaTeX is a lingua franca for technical communication in the Mathematics community. For other communities, leveraging the MS Office equation editors might be an attractive option. For active document settings (e.g. in semantic publishing systems like Planetary [Koh+11]), formulae might be instrumented with a "search similar formulae" interaction. The same holds for integrated semantic development system such as Mathematica.

[5] The demo is temporarily available at `http://arxivdemo.mathweb.org/index.php?p=/article/MWS`; we will provide a more permanent location for the final version of the paper.

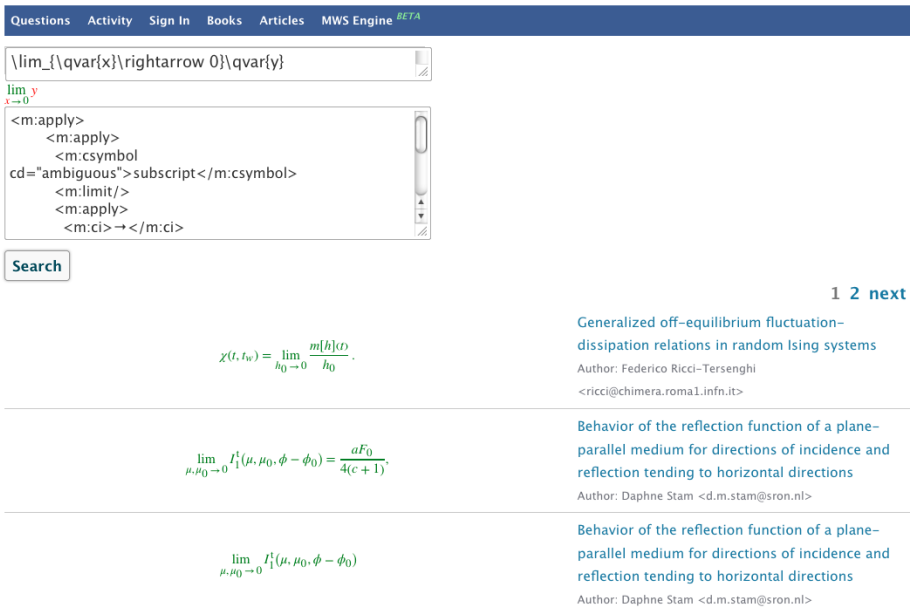


Fig. 5. MWS-0.5 arXivDemo Search Interface

4 Evaluation

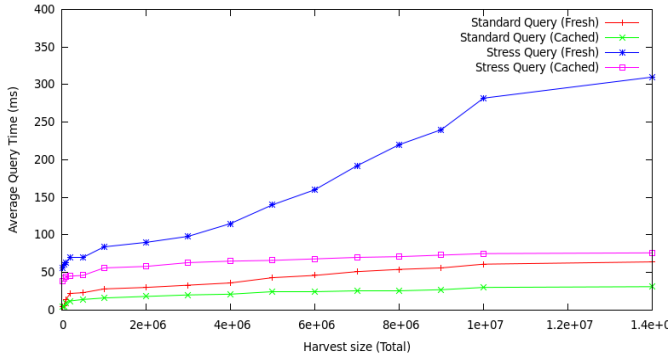
We evaluate the MathWebSearch implementation on a large corpus of mathematical formulae:

The arXMLiv Corpus Our group is working on the translation of the almost 750.000 TEX/LATEX articles on Physics, Mathematics, and Computer Science in the Cornell ePrint archive (see `http://www.arXiv.org`) to MathML [SK08]. **The arXMLiv corpus** is the result of translating ∼72% of the arXiv papers. For our evaluation we have harvested ca 65% (the fragment that have been converted without errors), resulting in a total of 115 million expressions. A trivial estimation suggests that the full arXMLiv corpus would contain approx. 245 million formulae. To harvest these, the arXMLiv crawler goes recursively through the pages of [arXMLiv] extracting the content MathML[6] elements, combines them with URI references, and reports them to MathWebSearch. We will now report on a performance analysis for MathWebSearch parametrized on **harvest size** (see Figure 6). As our index also indexes subformulae, we include them in the harvest size. Note that in the arXMLiv corpus a formula has 5.6 proper subformulae on average[7] so we estimate the number of indexable formula occurrences in the arXiv corpus to be $6.6 \times 245 \times 10^8$ or 1.6 billion. Note that

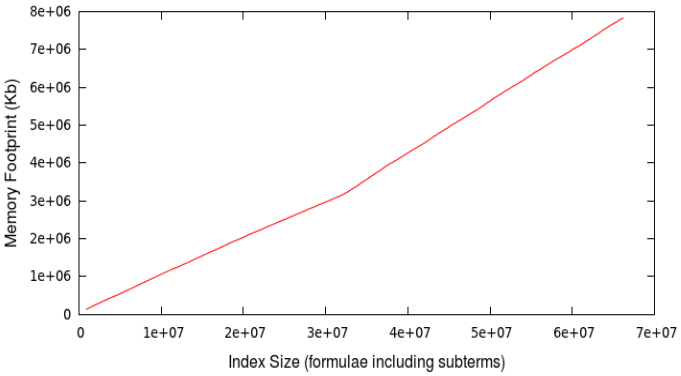
[6] The result of the transformation contains both Content and Presentation MathML representations in parallel markup.

[7] This rather low number comes from the fact that roughly 2/3 of the formulae in the arXMLiv corpus consist of only one letter; these are largely irrelevant for search purposes.

many of these formulae will actually be identical, leading to space savings in the index: recall that the URIs of the subformula occurrences are stored in a data base indexed by the leaves of the index (see Figure 3). Thus the index grows with the formula, whereas the database grows with the formula occurrences.



(a) Query Times



(b) Memory Usage

Fig. 6. Experimental Performance Analysis

Average query times We'll start with the time efficiency aspect, as this is highly relevant for a search system. The average query times [8] are presented in Figure 6(a). This experiment involves measuring several query response times. A standard query has answsize = 30 and limitmin < 9000. Response times are measured for standard queries (fresh and cached) from 0 up to 5 qvars. Additionally, stress queries with answsize = 10000 are used.

As one can see, the query response times are fairly constant as the harvest data increases. This fits with the theory, as the querying process will follow the same paths in the index (because the query data remains constant). The

[8] Note that the queries were sent from the local network, to eliminate any channel delays

small increase is due to the slightly higher density of the index tree (which affects retrieval of the right path). The gap between the fresh and cached stress queries is expected, due to the fact that the current bottleneck is retrieving the meta-data from the external database. Hence, caching significantly improves the results. In comparison to MathWebSearch 0.1 [KŞ06], which reported query times below 50 ms for simple queries and 200 ms for stress queries on a harvest size of 1.6 million, we see that the query times have not increased (even after normalization for hardware effects).

Memory usage The graph in Figure 6(b) presents the memory footprint of the mwsd process, as the system indexed 11.5 million expressions (67 million including subterms) harvested from the arXMLiv corpus. In comparison to MathWebSearch 0.1, which reported a memory footprint of 770 MB for a harvest size of 1.6 million, we see a space efficiency improvement by a factor of five. Contrary to our expectations of logarithmic increase, we see a fairly linear graph; the fact that the gradient became steeper after ∼33 million expressions is particularly unexpected. We are still investigating this.

Nevertheless, the experimental results are valuable to estimate the memory necessary to index the entire arXiv corpus. Assuming linear scaling across the 245 million formulae estimated for the arXiv, the memory necessary to index all the formulae would be $245 \times 8/11.5 \approx 170Gb$ according to our experiment. As this transcends the RAM of most machines, we have extended mwsd so that it can be distributed: a reasonable size computer cluster could easily accommodate the entire arXMLiv corpus and thus provide content-based formula search for `arXiv.org`.

5 Distributing MathWebSearch

We are currently implementing a distributed version of MathWebSearch. The core components, like the RESTful interface, the data formats, and the indexing data structures, remain the main building blocks. We complement them with a distributable version of our storage data structure, data persistency, migration and load balancing. Next, we will present some of the design decisions.

5.1 A Distributable Substitution Tree

As presented in Section 3, the main indexing data structure is a DFS substitution tree. To represent the tree in a manner that supports cross-machine links, we use three types of index nodes:

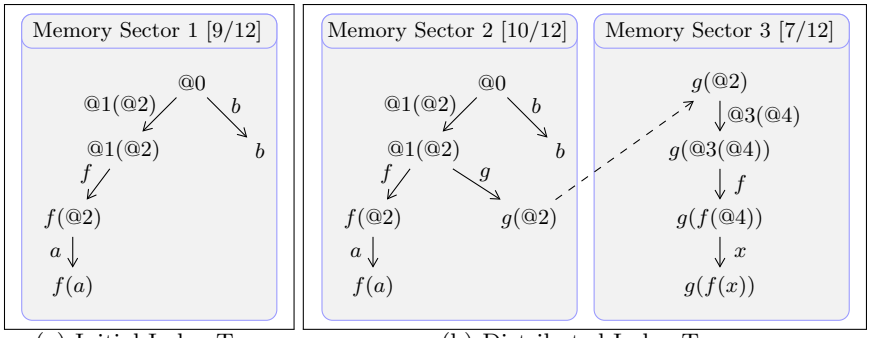
Internal Index Nodes are used to navigate through most parts of the tree. Their data stores mappings from token ID to the corresponding index node.

Leaf Index Nodes represent the end of a particular formula and its corresponding ID in the URL+URI database.

Remote Index Nodes represent cross-sector links. Their data consists of a pair of memory sector ID and node ID, which uniquely determine the corresponding index node.

As harvests are fed into the system, the index is built. Instead of building it on the heap (with no control over individual node's memory locations), the system places index nodes inside specific *memory sectors*.

Memory sectors are basically the smallest units of replication, migration, and load balancing. Each consist of a forest of index node trees packed into a continuous area of memory of fixed size[9]. Also, for each memory sector, we have an attached *local database* corresponding to the leaf index node results. As the index grows, individual sectors get filled with these trees. When a sector reaches a given threshold, two new sectors are created and the contents of the original sector get split between the new ones.



(a) Initial Index Tree (b) Distributed Index Tree

Fig. 7. Tree Distribution across Memory Sectors

In Figure 7, we present an example of term insertion which causes a tree split. Consider the initial tree containing the expressions $f(a)$ and b and memory sectors of size 12 units, as depicted in Figure 7(a). We consider a memory model where each link and each node costs 1 unit. Let's insert $g(f(x))$. As the resulting tree would overflow the current memory sector, a new one is created and parts of the tree are migrated to this new sector (See Figure 7(b)). To split the tree, the system performs a DFS traversal. As it traverses, the algorithm monitors two factors:

- the size of the already explored part of the tree (equivalent to the proportion of the split)
- the DFS fringe (each internal index node in here will generate remote index nodes).

We want to minimize the number of remote index nodes, while making the best balancing effort. In the current version, we choose the split which separates the

[9] The exact size depends on the RAM sizes of the nodes where the system will run, typical values ranging between 256Mb and 2048Mb.

trees with at most 40-60 bias, while minimizing the number of generated remote index nodes.

By representing the sectors as memory mapped files, persistency is achieved, since the contents of a sector can be easily dumped to, and loaded from secondary storage. Additionally, migration becomes trivial, since the persistent copy can be sent across the network, and re-mapped into another system's memory (of course, we assume endian-compatible machines).

Last but not least, by clustering the big index into continuous memory sectors, we can further reduce its size. By restricting sector sizes to 2 Gb, we can use 32bit relative pointers and further reduce the overall memory footprint of the tree. We have not been able to conduct large tests yet, however, since with 64bit pointers we could not determine any significant differences in memory consumption between the centralized and decentralized implementations, with the 32bit ones, we expect a memory reduction on the order of 40%.

5.2 Architectural Overview

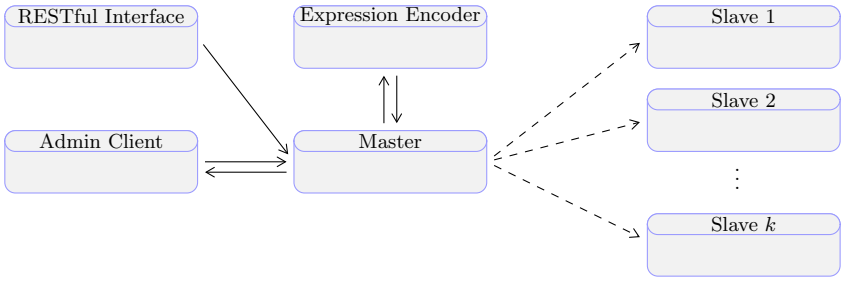


Fig. 8. Distributed Architecture - Communication Flow

Figure 8 portrays the major system components, as well as the communication flow around the cluster. There are a couple of central components listed on the figure, as well as some internal ones which are not, all of which to be explained in this section.

There are two main entry points for the application. The first one is the *RESTful Interface*, which like its centralized counterpart, is tasked with waiting on incoming requests (in the form of HTTP POSTs), reading and forwarding them. Moreover, it will also be in charge of replying to the connected client with the response to his request, once it is resolved. The *Expression Encoder* is an intermediary component that encodes the received input data (from REST requests) into the respective internal format (for either updates or queries) that is to be used throughout. This encoding is algorithmically similar to the centralized version, (see section 3.1), yet adapted for distributed communication. The second entry point in the application is through *admin client*s. These are entities that can be used to connect to the cluster and issue live commands as well as monitor state.

Since our architecture follows a Master-Slave communication model, the main coordination in the cluster is done by the Master Node. Its responsibility is to direct external requests, from either the RESTful interface or from admin clients, to the respective slave nodes that should handle them. Furthermore, it is also tasked with internally coordinating communication and work among the various slave nodes. As such, on cluster start-up, the master comes online, prepares the internal book-keeping (including the connection to the REST daemon) and proceeds to listen for incoming connections. These can be either slaves connecting to it to join the cluster setup, or admin clients. It is important to note that, the master will only handle overview tasks – above the actual memory sector level, so it will not impose a bottleneck.

Slaves, on the other hand, are in charge of serving, updating and querying across memory sectors. Each Slave is in charge of a partition of the memory sectors, on which it performs actions upon receiving messages. These can be data messages (update or query) or admin messages (like dropping or loading specific memory sectors). While updates and queries can originate from any node, admin messages are always sent from the master node. Upon coming online, slaves need runtime information to locate the master and connect to it. After establishing connection, the master issues them a unique machine ID that will represent them throughout the session, along with some extra information to help coordinate further communication.

For this, the master maintain the following internal metadata-structures:

Slave Map represents the mapping between assigned *Slave Machine IDs* to their addresses. This is used by slaves to directly connect to each other.

Memory Sector Distribution Map represents the mapping between assigned *Memory Sector IDs* to the slave machine IDs of the servants. This is used to resolve requests that need to go to a specific memory sector directly into a message to a machine.

Expression Root Map represents the mapping between an encoded expression token ID to the memory sector id of its container[10]. This is used by the master to resolve a request on a given expression directly to a memory sector that it will be found on.

Split Tree Record represents the history of splits for all the memory sectors in the system, as well as data about the designated remote index node IDs. It maps each remote index node ID to the corresponding memory sector. This is used by the slave nodes to resolve remote index node jumps to the subsequent memory sectors. Also, this is lively updated throughout the cluster in case of memory sector splits.

Some of this metadata is also copied to the slaves: The Slave Map and Memory Sector Distribution Map are fully replicated, as slaves need to resolve requests pertaining to memory sectors that they might not be responsible for. Furthermore, the part of the Split Tree Record that related to the memory sectors a slave is responsible for is also replicated on each respective slave, for the same

[10] The memory sector forest contains trees starting from these expression roots

resolution reasons. The Expression Root Map is only used by the master to direct encoded requests (and it is modified on every update going towards memory sectors).

Another important observation is that, once a master-slave connection is established, this enables a two way communication protocol. Through this, the master is allowed to send messages to the slave at any given time (such as query or update requests or status notices), while the slave is responsible to periodically check-in with the master (currently, every 60 seconds) via heartbeats. For each of these heart-beats, the master will lazily[11] ask the slave to update their internal mappings, should it be necessary. If a slave fails to respond within the allotted time, it will be assumed dead by the master and thus it and all mapping entries that would lead to it or its memory sectors will be invalidated.

Distributed Updates In the current setup, the only way to load the main index of the system is via admin commands. Through this, the master takes in an MWS harvest file and proceeds to read and parse the expressions. For each one, the Expression Encoder is used to transform it into internal representation. Afterwards, if the Expression Root already exists in some memory sector, the prepared insertable item is forwarded to the respective machine that is responsible for it, by performing a lookup on the Expression Root Map and subsequently on the Memory Sector Distribution Map. If the Expression Root did not exist, than it will be assigned to a memory sector in a Round Robin fashion and then the request will be forwarded. Finally, once the request reaches the respective memory sector, a DFS of the tree starting at the Expression Root will end in two possible ways. Either a leaf index node is reached and a database insertion will be triggered, or a remote index node will be discovered, case in which, after mapping resolution, the remaining request will be forwarded to the respective machine and the algorithm recurses there.

Distributed Querying As for queries, upon receiving a request $\mathcal{Q}$, the RESTful interface assigns it an unique ID and passes it on to the Expression Encoder and then onto the master. From here, the same resolution used for insertion is employed to find the slave which serves the memory sector containing the root of the query expression. Here, the query is first searched in the slave cache. In case of a hit, the response is sent back to the RESTful interface, which uses the assigned ID to reply to the client. If the query misses the cache, the slave node initiates and coordinates the internal searching process. The expression look up starts within the memory sector. Whenever a remote index node is reached, the query is forwarded to the respective memory sector. However, when a leaf index node is reached, the respective solutions are fetched from the database, forwarded to the coordinating slave node and cached there. An individual search process is stopped either when there are no more pending remote queries, or when

[11] The lazy updates allow us to not worry about invalid connections or two-way communication requests; if messages need to be routed to machines that do not have an address cached, the requests are just dropped.

a set limit of results is reached. To increase performance, remote requests are handled asynchronously, results are sent directly to the coordinator (the slave which initiated it) and queries are processed in bulks[12].

6 Conclusion and Future Work

We have presented a scalable extension of a search engine for mathematical formulae. In contrast to other approaches, MathWebSearch uses the full content structure of formulae and is easily extensible to other content-oriented formats. Our first evaluation shows that query times are low and essentially constant in index/harvest size, so that a search engine can scale up to web proportions. Contrary to our expectations, index size is linear in harvest size for the arXiv corpus, which transcends the main memory limits of standard servers. Therefore, developing parallelization/distribution strategies is a priority. This paper reports the establishment of the core distribution algorithms and functionality and shows viability of the approach in principle. Exploring the distribution, management, and load balancing of a distributed cluster of search nodes is beyond the scope of this paper.

Even though based on standard practices from distributed systems, the system architecture presented here is tailored to the case of distributed substitution indexes and unification-based queries. Standard database distribution techniques do not seem to be applicable, since there indexes essentially contain metadata about locating or accessing data efficiently (and are thus orders of magnitude smaller than the data itself), whereas the substitution tree index is basically an organization of the data (formulae) itself optimized for sharing. Generic database features, such as ACID properties, which might benefit from database distribution techniques are not currently in the scope of this system.

We conclude the paper with a tabulation of open research areas for information retrieval in mathematical/technical documents.

6.1 Additional Corpora

The arXiv corpus we are currently using for benchmarking is paradigmatic for the "informal but rigorous mathematics" that dominates mathematical communication today. Here, the Content MathML has to be heuristically reconstructed from the presentation in the sources. This is unnecessary for corpora of formalized mathematics, e.g. the Mizar Mathematical Library [MizLib] with over 50 000 formal theorems. The problems of obtaining the content MathML are different here: Even though the representations are formal in principle, the surface languages are human-oriented, and fully explicit representations need reconstruction processes (e.g. for reconstructing elided types and arguments, resolution of operator overloading, etc.). We are currently working on Content

[12] Although the client only requests 30 items, up to 1000 may be retrieved internally and placed into the cache.

MathML exporters for the Mizar Library and the TPTP (Thousands of Problems for Theorem Proving) library [SS]. Other future targets could be the input files of mathematical software systems e.g. computer algebra systems like Mathematica, numerical computation systems like MatLab, or statistics programs like the R system [Tee11].

6.2 Extending the Indexing

A current weakness of the system is the fact that it can only search for formulae that match the query terms up to α-equivalence. Many applications would for instance benefit from stronger equalities. Our search in the running example might be used to find a useful identity for $\int_0^\infty f(x) \cdot g(x)dx$, if we know that $s(x) \cdot s(x) = s^2(x)$. MathWebSearch can be extended to an *E-Retrieval* engine (i.e., search modulo an equational theory E or logical equivalence) without compromising efficiency by simply E-normalizing index and query terms (see [NK07] for a first implementation).

In the long run, we plan to extend MathWebSearch, so that it can take more document context information into account, i.e., not just keywords from the text around the formulae but e.g. the topology of theories in the OMDoc format: It would be very useful, if we could restrict searches to formulae that are consistent with current (mathematical) assumptions.

6.3 Result Ranking

Advances in ranking have made word-based search engines scalable from a user point of view. For formula search engines ranking is an open research question, there is only one paper that covers this in a more presentation-search oriented setting [You06a]. To solve the problem, we have to consider the following aspects: 1. What is a good measure for relevance in theory (pagerank only applies to pages)? 2. How can be compute this efficiently. 3. Can we organize the index, so that it finds the most relevant hits (as estimated by this measure) first? 4. Is a single measure enough? We plan to look at the following simple measures as starting points: 1. pagerank by citations over the papers 2. the size (whatever that means) of the substitutions (small might be beautiful) 3. similarly, the size of the formulae 4. popularity of the papers (by download) Finally, we would like to allow specification of content queries using more widely known formats, like LaTeX: strings like `\frac{1}{x^2}` or `1/x^2` could be processed as well. This can be reached by applying an extension (by query variables) of the LaTeX to XML transformation used on the arXiv to process queries. The new LaTeXML daemon [GSK11] allows integrating this efficiently.

6.4 Advanced Search Services

Another important application of the unification search in MathWebSearch is applicable theorem search (like our example with Hölder's inequality in Section 2). The MathWebSearch system already supports the necessary queries

(unification), but the arXiv corpus we are currently using does not have the necessary degree of formalization (explicitly marked up universal quantifications). We plan to utilize (possibly shallow) linguistic technologies to reliably analyze phrases like "*Let f and g be functions from $\mathbb{R}$ to $\mathbb{R}$...*" that mark the identifiers f and g as universal and to retrieve the associated sortal restrictions. Note that the linguistic capabilities of the variable spotter have to be considerable to detect the difference between "*... where c is a natural number*" and "*... where x is the number between 1 and n, such that...*" (only is c universal) or to detect that universals in a negative scope are indeed existential.

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