

Large-scale proof and libraries in Isabelle/HOL

Gerwin Klein



Australian Government

Department of Communications,
Information Technology and the Arts

Australian Research Council

NICTA Members



Department of State and
Regional Development



The University of Sydney



Queensland University of Technology



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AUSTRALIA

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L4 **Verified**

L4-Verified

1 microkernel

8,700 lines of C

0 bugs*

qed

*conditions apply

Windows

An exception 06 has occurred at 0028:C11B3ADC in VxD DiskTSD(03) + 00001660. This was called from 0028:C11B40C8 in VxD voltrack(04) + 00000000. It may be possible to continue normally.

- * Press any key to attempt to continue.
- * Press CTRL+ALT+RESET to restart your computer. You will lose any unsaved information in all applications.

Press any key to continue



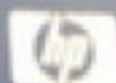
Windows Vista

Stunning. Breakthrough. Entertaining.

HP TouchSmart PC and Microsoft Windows Vista deliver you a PC experience designed to fit wherever life happens.

Innovative solutions brought to you by:

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BEST
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The Problem



Small Kernels

Small trustworthy foundation

- hypervisor, microkernel, nano-kernel, virtual machine, separation kernel, exokernel ...
- High assurance components in presence of other components

seL4 API:

- IPC
- Threads
- VM
- IRQ
- Capabilities

Untrusted

Legacy
Apps

Linux
Server

Trusted

Sensitive
App

Trusted
Service

Hardware

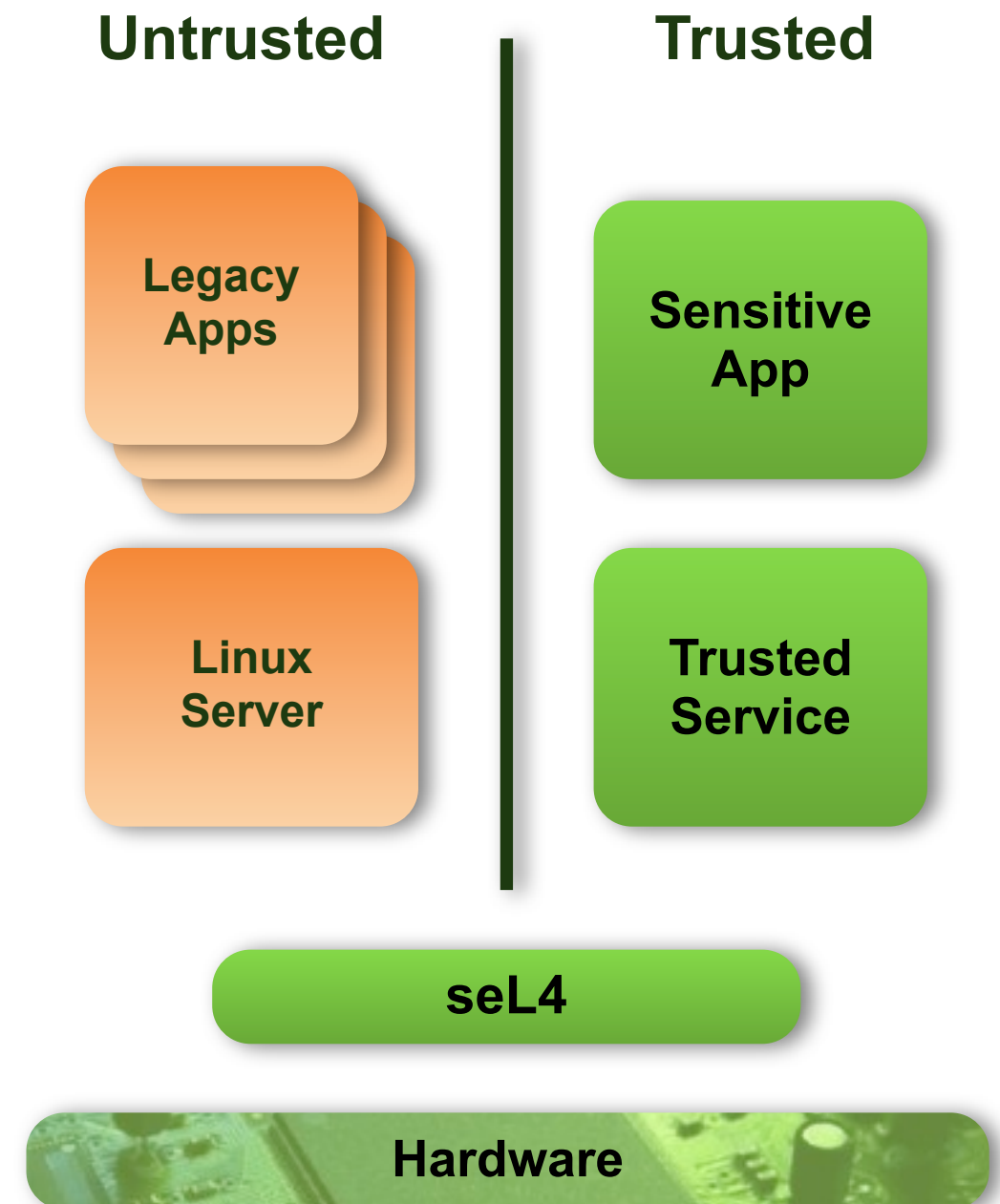
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seL4 API:

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The Proof

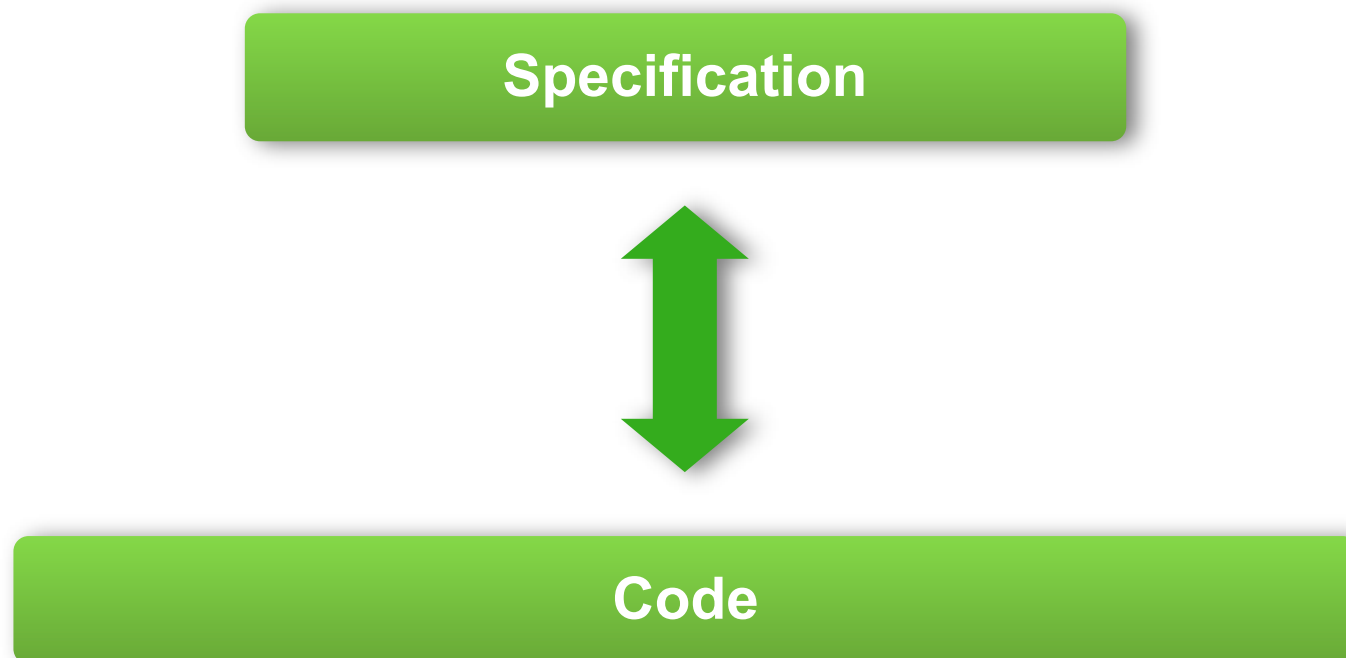


The Proof



Functional Correctness

Proof



Functional Correctness

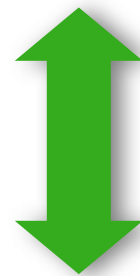
What

Specification

definition

```
schedule :: unit s_monad where
schedule ≡ do
  threads ← allActiveTCBs;
  thread ← select threads;
  switch_to_thread thread
od
OR switch_to_idle_thread
```

Proof



Code

Functional Correctness



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Proof

How

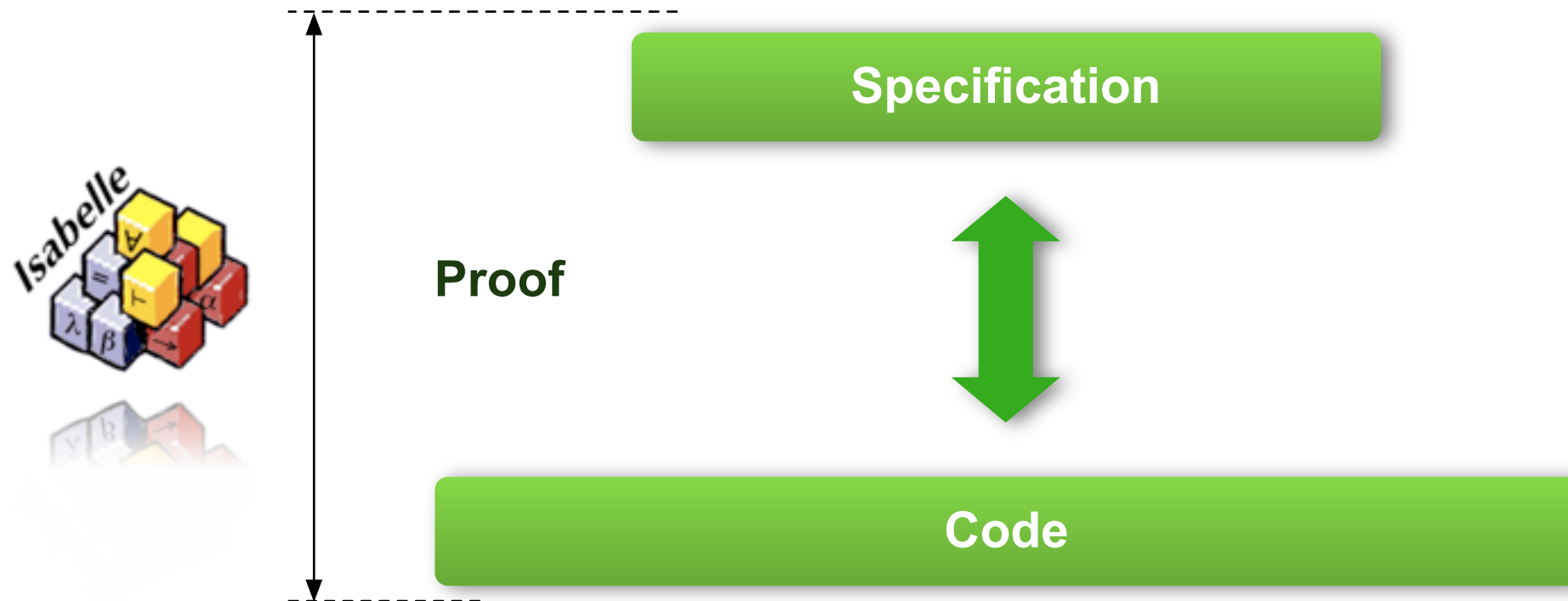
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void
schedule(void) {
  switch ((word_t)ksSchedulerAction) {
    case (word_t)SchedulerAction_ResumeCurrentThread:
      break;

    case (word_t)SchedulerAction_ChooseNewThread:
      chooseThread();
      ksSchedulerAction = SchedulerAction_ResumeCurrentThread;
      break;

    default: /* SwitchToThread */
      switchToThread(ksSchedulerAction);
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void
chooseThread(void) {
  prio_t prio;
  tcb_t *thread, *next;
```

*conditions apply



***conditions apply**



Expectation

Specification

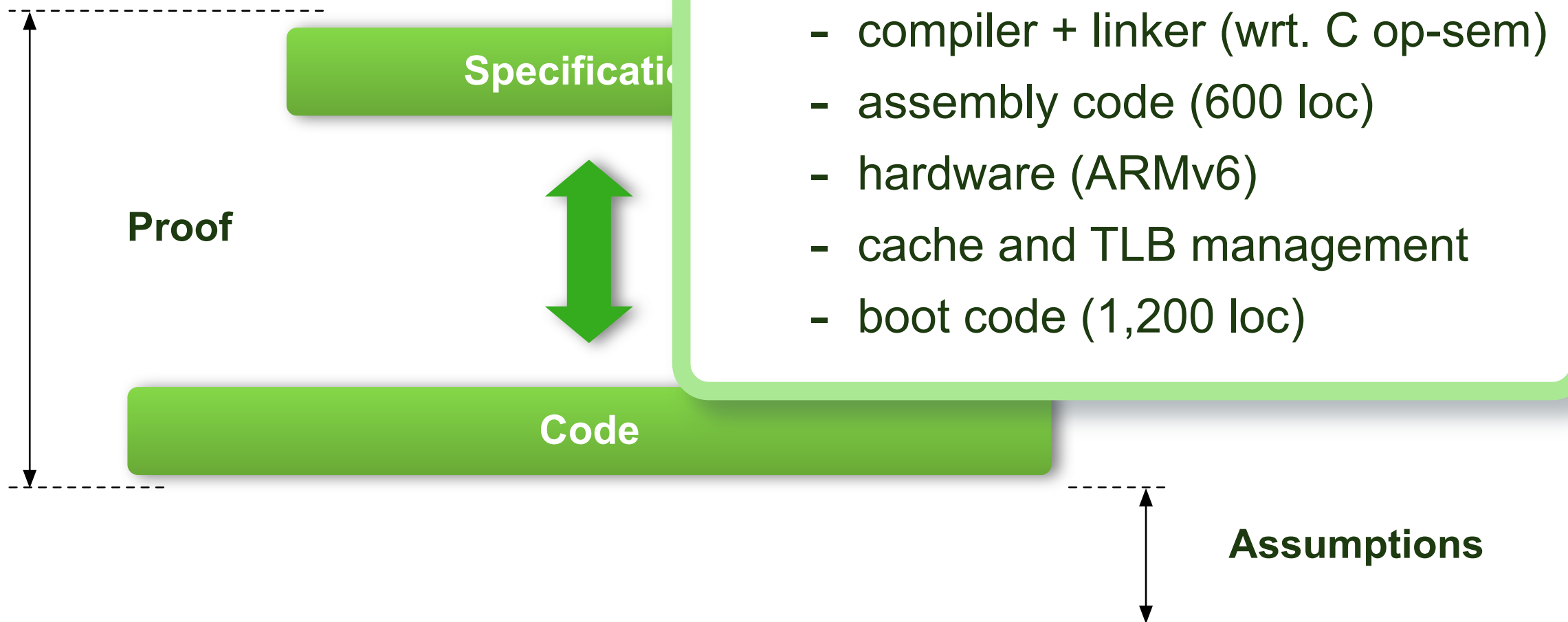
Proof

Code

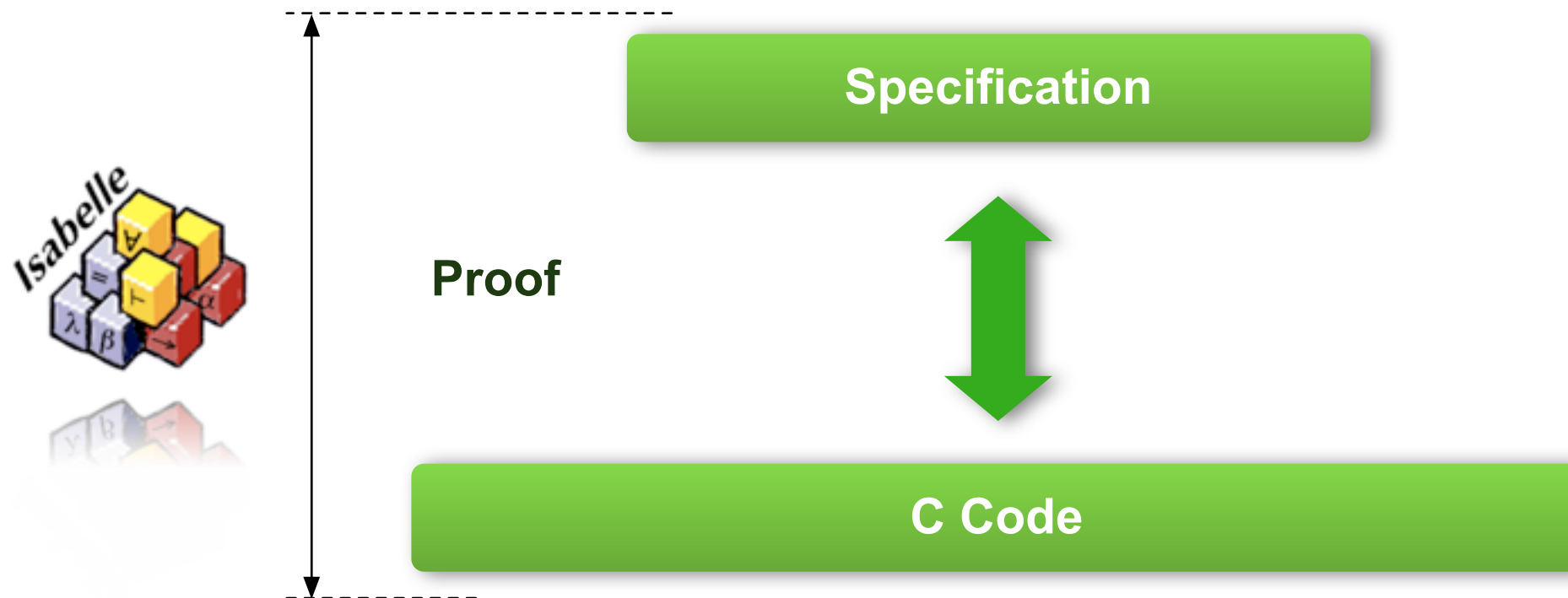
Assumptions



***conditions apply**

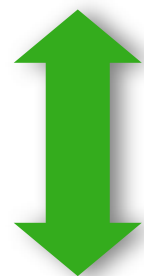


Proof Architecture

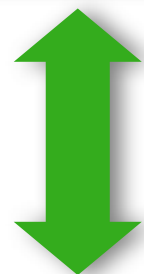


Proof Architecture

Specification

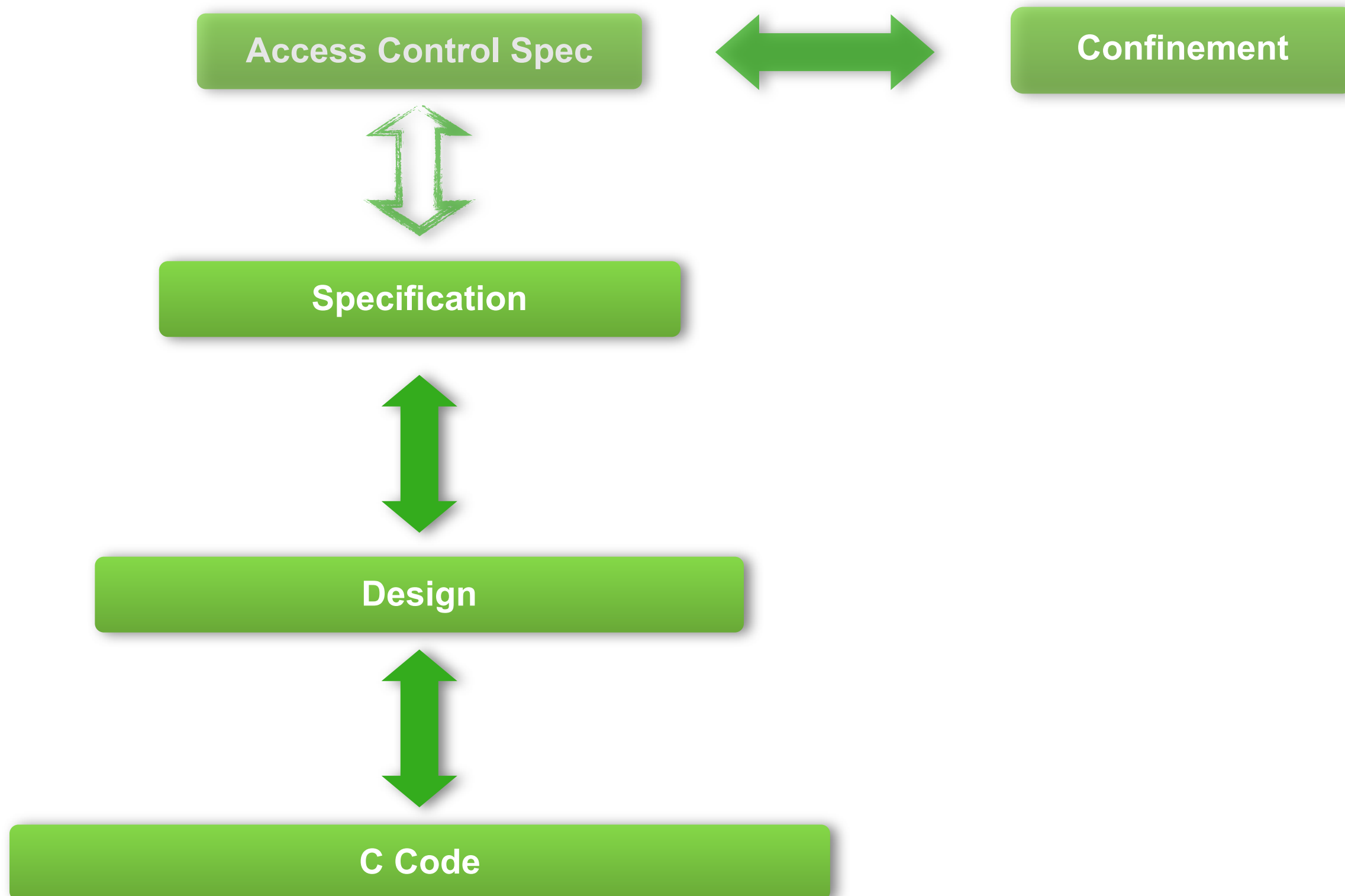


Design

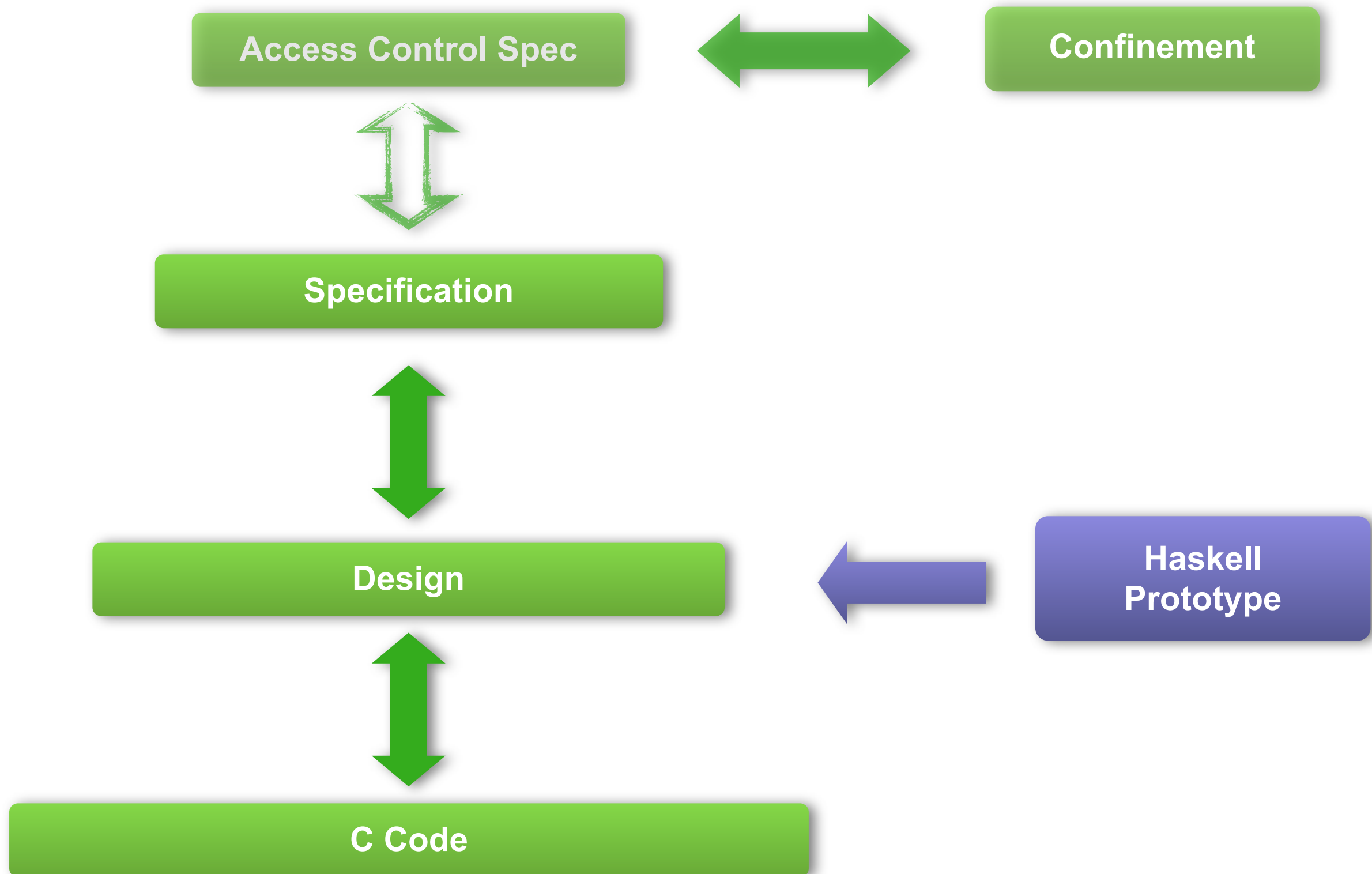


C Code

Proof Architecture



Proof Architecture



Proof Architecture

Access Control Spec

Confinement

Specification

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Design

Haskell
Prototype

C Code

Proof Architecture

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Design

C Code

```
schedule :: Kernel ()
schedule = do
  action <- getSchedulerAction
  case action of
    ResumeCurrentThread -> return ()
    ChooseNewThread -> do
      chooseThread
      setSchedulerAction ResumeCurrentThread
    SwitchToThread t -> do
      switchToThread t
      setSchedulerAction ResumeCurrentThread

chooseThread :: Kernel ()
chooseThread = do
  r <- findM chooseThread' (reverse [minBound .. maxBound])
  when (r == Nothing) $ switchToIdleThread
  where
```

Proof Architecture



Access Control Spec

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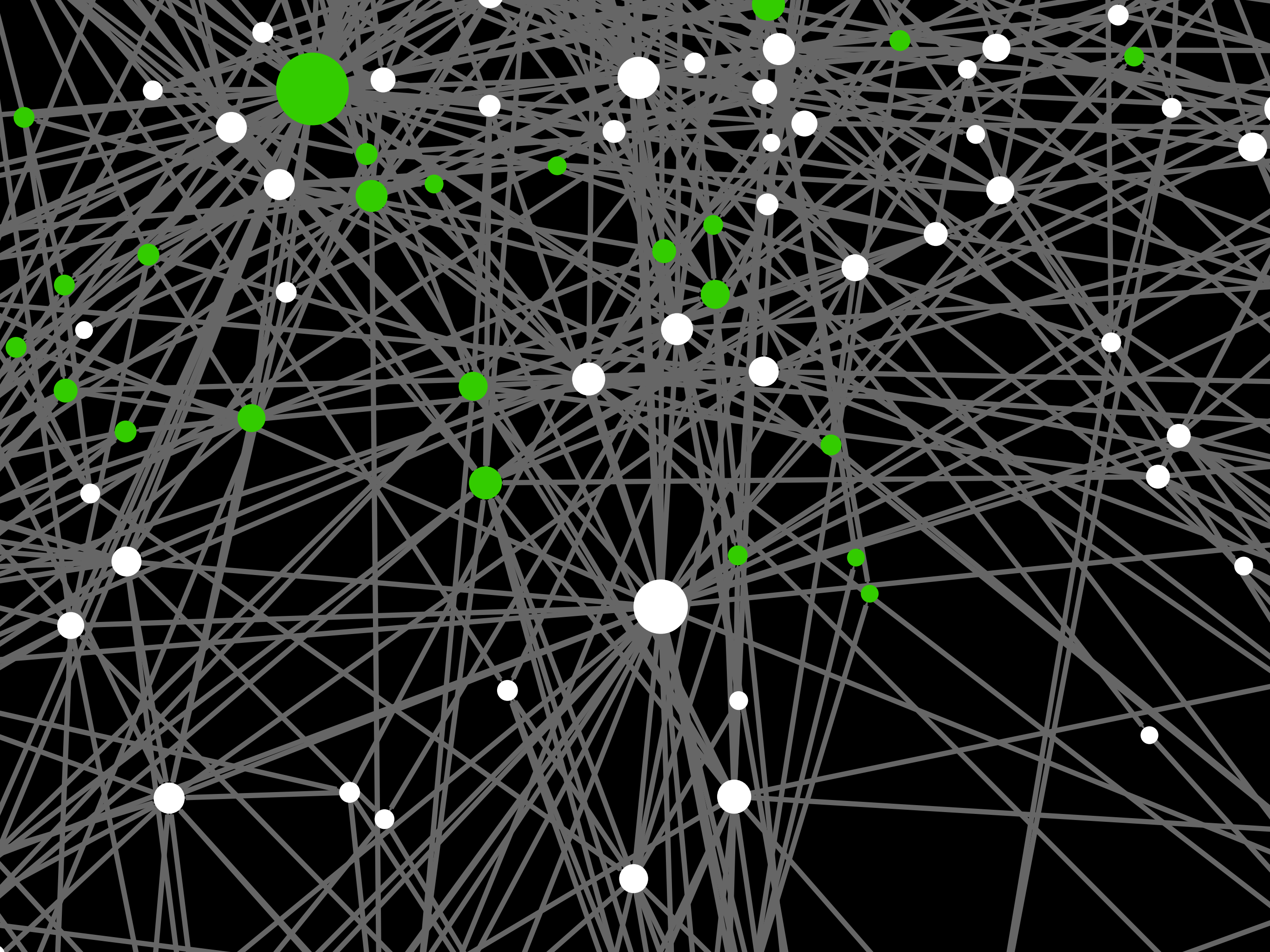
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seL4

Did you find any Bugs?



Bugs found

- in C: 160
- in design: ~150
- in spec: ~150

460 bugs

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Effort

Haskell design	2 py
First C impl.	2 weeks
Debugging/Testing	2 months
Kernel verification	12 py
Formal frameworks	10 py
Total	25 py

```
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chooseThr
prio_
tcb_t

for(p
f

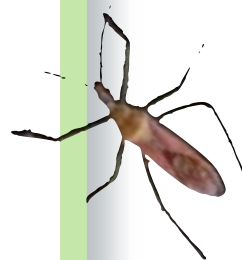
if(!isRunnable(thread)) {
    next = thread->tcbSchedNext;
    tcbSchedDequeue(thread);
}
else {
    switchToThread(thread);
    return;
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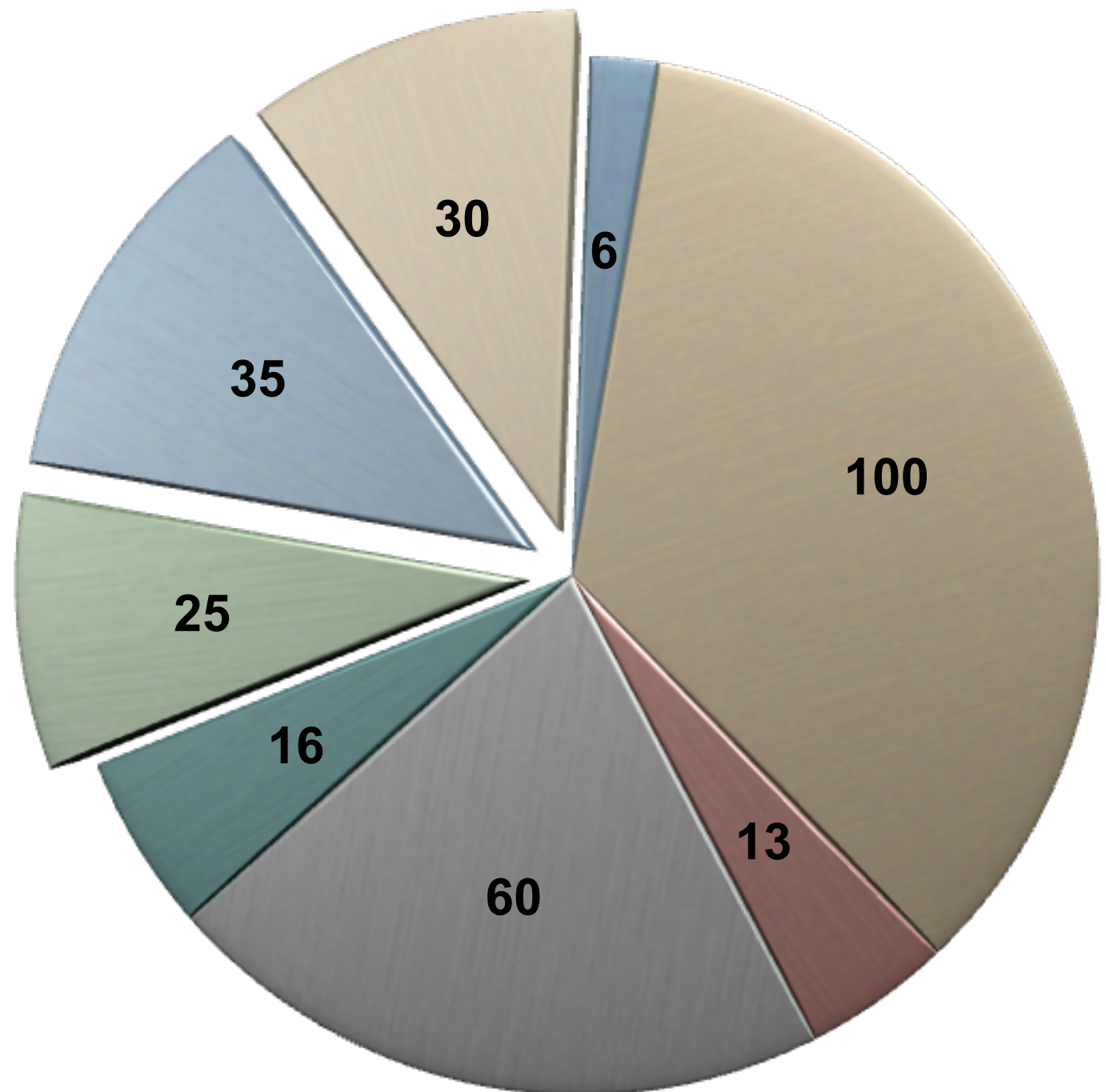
Proofs and Libraries



Proofs and Libraries



Main Proof Components



Theory Inclusion Graph



Theory Inclusion Graph



Main Libraries

- Libraries

- Word library
- Enums
- Haskell library support
- Monads
- Monad VCG + case splitter + strengthening
- Refinement on Monads
- Submonads and rewriting under refinement
- Refinement between Monads and C + tools
- SIMPL + imperative VCG
- Crunch
- LemmaBucket



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Tools

- **Productivity tools**
 - theorem search (find_thms)
 - theorem moving (Levity)
 - finding definitions (locate)
 - theorem web search (www_find)
 - numeral syntax (hex, oct, binary)
- **Isabelle**
 - Record package
 - Proof cache
 - attributes like “rotated”
 - automated simp rules for case construct



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What we thought would work

- Automation
 - First order provers (vampire)
 - SAT for word problems
 - Termination proofs
- What we thought might be a problem
 - Performance
 - Memory
 - Lemmas/goals/proofs too big to parse

Problems



Problems



- What turned out to be a problem

Problems



- What turned out to be a problem
 - Isabelle updates

Problems



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 - Performance, but not as bad

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 - Annoying word proofs
 - Developing/maintaining libraries + frameworks
 - Hard to get over good enough

Wish List



- Better name space management
- Better performance (time + memory)
- Better parallelisation
 - run nitpick, quickcheck, sledgehammer etc in background
 - run proofs faster interactively
- Better dependency handling
 - faster turnaround for deep definition changes
 - faster image rebuilding

Archive of Formal Proofs



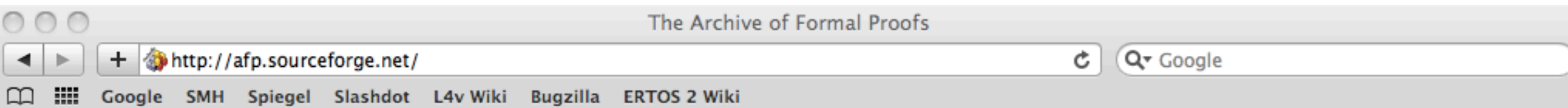
Archive of Formal Proofs



- Repository of Isabelle proofs
 - Open Source (BSD + LGPL)
 - Maintained
 - Archived, version controlled
 - Quality controlled
- Hosted on SourceForge
 - [\[http://afp.sf.net\]](http://afp.sf.net)



Screen Shot



THE ARCHIVE OF FORMAL PROOFS

The Archive of Formal Proofs is a collection of proof libraries, examples, and larger scientific developments, mechanically checked in the theorem prover [Isabelle](#). It is organized in the way of a scientific journal and has an ISSN: 2150-914x. Submissions are refereed. The preferred citation style is available [\[here\]](#). A [development version](#) of the archive is available as well.

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2010

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Author: Joachim Breitner

2010-06-20: [Category Theory](#)

Author: Alexander Katovsky

2010-06-17: [Executable Matrix Operations on Matrices of Arbitrary Dimensions](#)

Author: Christian Sternagel and René Thiemann

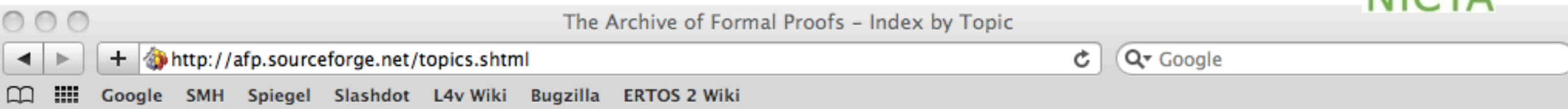
2010-06-14: [Abstract Rewriting](#)

Author: Christian Sternagel and René Thiemann

2010-05-28: [Verification of the Deutsch-Schorr-Waite Graph Marking Algorithm using Data Refinement](#)

Author: [Viorel Preteasa](#) and [Ralph-Johan Back](#)

Index by Topic



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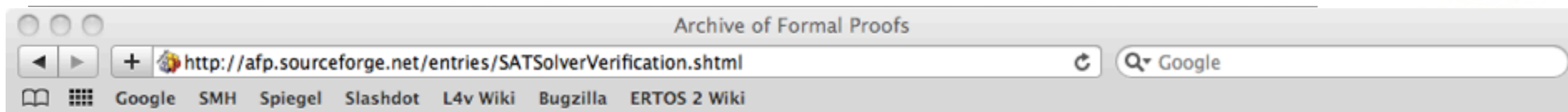
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An Entry



FORMAL VERIFICATION OF MODERN SAT SOLVERS

Title:	Formal Verification of Modern SAT Solvers
Author:	Filip Maric
Submission date:	2008-07-23
Abstract:	This document contains formal correctness proofs of modern SAT solvers. Following (Krstic et al, 2007) and (Nieuwenhuis et al., 2006), solvers are described using state-transition systems. Several different SAT solver descriptions are given and their partial correctness and termination is proved. These include: • a solver based on classical DPLL procedure (using only a backtrack-search with unit propagation), • a very general solver with backjumping and learning (similar to the description given in (Nieuwenhuis et al., 2006)), and • a solver with a specific conflict analysis algorithm (similar to the description given in (Krstic et al., 2007)). Within the SAT solver correctness proofs, a large number of lemmas about propositional logic and CNF formulae are proved. This theory is self-contained and could be used for further exploring of properties of CNF based SAT algorithms.

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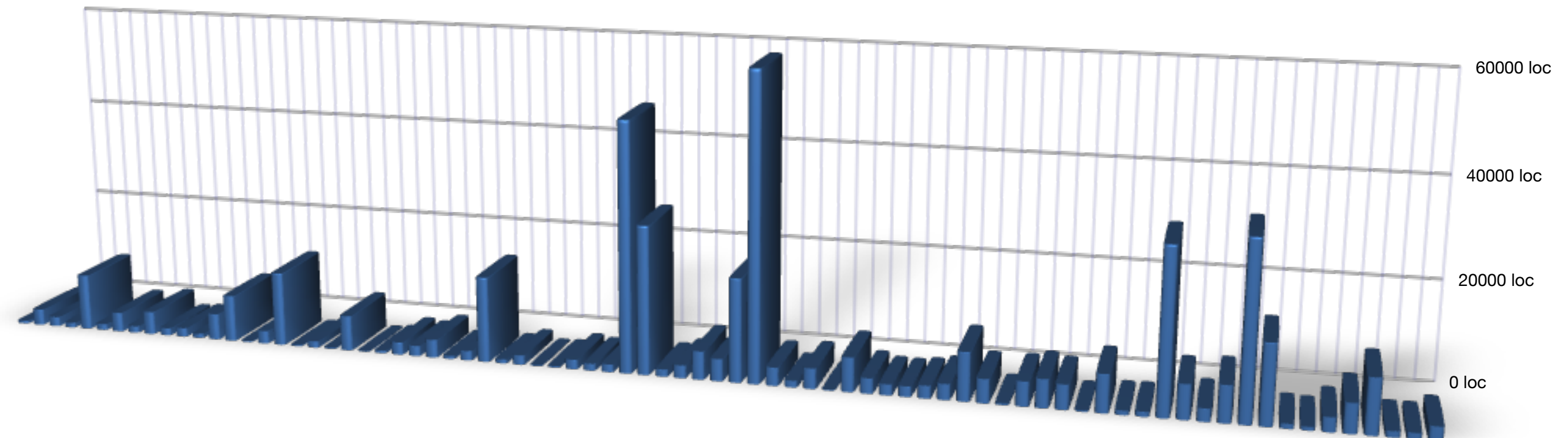
Statistics

- Numbers

- started 2003
- now 77 entries
- 473 kloc of proof scripts
- 22,000 lemmas
- runs 4-6h daily

- Typical entry

- belongs to a paper
- nice proof document
- small, 1-4 kloc
- some big developments



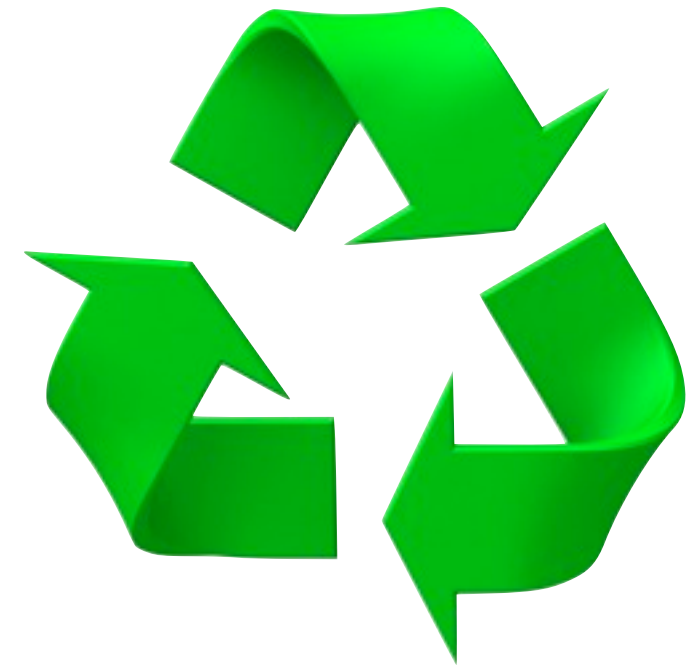
What works well

- **Archival and Maintenance**
 - everything kept up to date by Isabelle team
 - new release with every Isabelle release
 - good number of submissions
 - mostly good quality submissions
 - some authors keep improving their entries
 - (but need better high-level change tracking, versioning)
 - good test bed for Isabelle team
 - good place to publish proofs for paper authors
 - authors can get repository access



Reuse?

- Not much reuse:
 - Few entries designed as libraries
 - Few entries used in others (3)
- Reasons
 - Cultural (mostly papers)
 - Good libraries go directly into Isabelle distribution
 - Technical hurdles
 - not enough instruction on how to include
 - could give more built-in support, name spaces again
 - can't import from more than one image
 - dependencies? hackage site?



Summary



Summary



Proof Libraries are like Program Libraries

- **L4.verified:**
 - 200kloc proof
 - lots of libraries
 - some are reusable, costs more work, usually worth it
- **Archive of Formal Proofs**
 - 77 entries, 470kloc proof in total
 - works well for papers
 - not so well for libraries yet, hope to get more in the future

Thank You

