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Survey on Mathematical Notations

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Survey on Mathematical Notations

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Summary

This paper aims at collecting various examples that illustrate the ambiguity of notations. In particular, we point out alternative notations that are used to present the same mathematical concept, i.e. mathematical content expression with the same meaning. In addition, we exemplify mathematical expression, which represent different mathematical concepts, but are presented with the same notation. Moreover, this work aims at classifying alternative notations according to the context their are used in.

Contents

1 Introduction

Wikipedia says: A *mathematical notation* is a writing system (in fact, a formal language) used for presenting concepts in mathematics or other fields. The notation uses *symbols* or *symbolic expressions* which are intended to have a precise *semantic meaning* (cf. [Wik08b]).

Notations are central for understanding mathematical discourse. Readers would like to read notations that transport the meaning well and prefer notations that are familiar to them. Therefore, authors optimize the choice of notations with respect to these two criteria, while at the same time trying to remain consistent over the document and their own prior publications. The latter task is indeed not trivial because different mathematical expressions may share the same notations. In contrast, the same meaning can be expressed by various alternative notations. Moreover, notations can be provided with different level of completeness, e.g. authors may choose to omit types and brackets. For example, $ax+y$ is actually $(ax)+y$, since multiplication "binds stronger" than addition. This may be clear to some readers, but not to all.

The markup of the meaning (or the content of a mathematical expression) and the used notations (the presentation for the mathematical expression) has been the first step to addressing the ambiguity of mathematical notations. Content Markup formats for mathematics such as OpenMath [BCC+04] and content MathML [ABC+03] concentrate on the functional structure of mathematical formulae, thus allowing mathematical software systems to exchange mathematical objects and to identify the meaning of a mathematical expression. For communication with humans, Presentation Formats such as Presentation-MathML [ABC+03] allow to specify alternative notation of the same content expression.

Various researchers have provided examples, in which mathematical expressions are differently presented depending on the *context* they are used:

In [SW06] Watt and Smirnova introduce possible *reasons* for multiple notations of the same mathematical concept, namely *area of application*, *national conventions*, *level of sophistication*, the *mathematical context*, and the *historical period*. In contrast to the five *"reasons"* in [SW06], the ActiveMath [MLUM06] distinguishes four *"context categories"* that influence the adaptation of notations, namely *language*, *different patterns of the argument*, the *author's style*, and *notations of the same collection*. In [KMR08] the notion of context dimensions and context values was used to describe the contextualization of notations.

This paper aims at collecting various examples that illustrate the ambiguity of notations. In particular, we point out alternative notations that are used to present the same mathematical concept, i.e. mathematical content expression with the same meaning. In addition, we exemplify mathematical expression, which represent different mathematical concepts, but are presented with the same notation. Moreover, this work aims at classifying alternative notations according to the context their are used in, in particular, by grouping them according to the dimensions *display style*, *level of expertise*, *language*, *individual styles*, and *area of application*. The illustrated examples are collected from various online sources as well as the course material of the General Computer Science lecture (GenCS) at the Jacobs University Bremen.

2 Display Style

Authors select notations for a specific output format:

Text Output	Display Output
$\sum_{k=1}^{n}$	$$\sum_{x=1}^{n} x$$
$\bigcup_{k=1}^{n}$	$$\bigcup_{k=1}^{n}$$

3 Level of Expertise

Mathematicians gloss over parts of the formulae, e.g., leaving out arguments, if they are non-essential, conventionalized or can be deduced from the context. Indeed this is part of what makes mathematics so hard to read for beginners, but also what makes mathematical language so efficient for the initiates.

In this section we collect examples that illustrate notations that can be classified based on different *level of expertise* of readers and authors:

School Level While $a \div b$ is mostly used in elementary school, $\frac{a}{b}$ is used in higher education [SW06].

Logarithm $\log(x)$ or even $\log x$ or $\lg x$ can be used for $\log_{10}(x)$ (cf. [KMR08])

Natural Logarithm denoted by either $\ln y$ or $\log_e y$. Teacher may choose to use the latter notation to lead to the former.

Negation Experienced users refer to $x^2-3yx+1$ rather than $x^2+(-3y)x+1$ (cf. [NW01])

Sinus Experienced users refer to $\sin^y x$ rather than $(\sin x)^y$

Multiplication Experienced users omit the times operator, if it can be deduced from the context: ab rather than $a * b$

3.1 Bracket Elision

Authors may choose to insert brackets that are not necessarily needed or to omit them if their readers are more experienced:

- $(a + b) + c = a + (b + c)$
- $ax + y$ is actually $(ax) + y$, since multiplication "binds stronger" than addition
- $5 + x * y^{n+3}$ rather than $5 + (x * (y^{n+3}))$

3.2 Type Elision

Authors may choose to leave out types, if they can be deduced from the context:

- $[\![t]\!]$ for $[\![t]\!]^{\phi}_{\mathcal{M}}$, if there is only one model $\mathcal{M}$ in the context and ϕ is the most salient variable assignment (cf. [KMR08])
- $\Lambda^* = \lambda FGX_\iota.F(X)\Lambda G(X)$ rather than $\Lambda^*_{(\iota\to o)\to((\iota\to o)\to(\iota\to o))} = \lambda F_{\iota\to o}G_{\iota\to o}X_l.F_{\iota\to o}(X_\iota)\Lambda_{o\to(o\to o)}G_{\iota\to o}(X_\iota)$

4 Individual Style

Depending on their practice, professors are using different combinations of (1) $A \subset B$, (2) $A \subseteq B$, and (3) $A \subsetneq B$, in which cases the presentation (1) represent different meanings: When using (1) and (2), (1) means that A is the proper subset of B (excluding the equality of both sets). In contrast, when using (1) and (3), (1) represents the subset of B (including the equality of both sets).

Alternatively, for (2) also $A \subseteqq B$ is used and for (3) one can also use $A \subsetneq B$, $A \subsetneqq B$, or $A \varsubsetneqq B$.

5 Language and Cultural Differences

This section gathers example that exemplify the cultural or language-dependent influence on notations.

Half-open Interval The *anglo-saxon* $(0, 1]$ and the *french* $]0, 1]$ (cf. [DL08])

Decimal Point is a symbol in arithmetics, which is used to separate the fractional part of a decimal from the whole part. For decimal numbers, *German* use a comma where *English* use a decimal point. For example $4,5$ in German is equivalent to 4.5 in English.

In contrast, in German 1000 is written/printed as either 1000, 1.000 or 1000 — using a *decimal point* (*Punkt*) or a *space* where English uses a comma $(1,000)$. This also applies to all German numbers above $1,000$.

Binomial Coefficient We can distinguish notations that are commonly used in different language. For example, various notations exists to denote the binomial coefficient, the number of k-element subsets of an n-element set $\frac{n!}{k!(n-k!)}$: The symbol is denoted with $\mathcal{C}^k_n$ in French/ Russian speaking countries and with $\binom{n}{k}$ in German/ English speaking countries. Further notation exists, e.g., $\mathcal{C}(n, k)$, ${}_n\mathcal{C}^k$, ${}^n\mathcal{C}_k$, ${}_n\mathcal{C}_k$, as well as system-specific notation, such as $binomial(n, k)$ in MAPLE and $Binomial[n, k]$ in MATHEMATICA [NW01].

Polish Notation for Sentential Logic Differences between Conventional and Polish Notation: The table below shows the core of Jan Łukasiewicz's notation for sentential logic. The "conventional" notation did not become so until the 1970s and 80s (cf. [Wik07a, Łuk67])

Mathematical Concept	Conventional Notation	Polish Notation
Negation	$\neg\varphi$	$N\varphi$
Conjunction	$\varphi \wedge \psi$	$K\varphi\psi$
Disjunction	$\varphi \vee \psi$	$A\varphi\psi$
Material conditional	$\varphi \to \psi$	$C\varphi\psi$
Biconditional	$\varphi \leftrightarrow \psi$	$E\varphi\psi$
Sheffer stroke	$\varphi \mid \psi$	$D\varphi\psi$
Possibility	$\Diamond\varphi$	$M\varphi$
Necessity	$\Box\varphi$	$L\varphi$
Universal Quantifier	$\forall\varphi$	$\Pi\varphi$
Existential Quantifier	$\exists\varphi$	$\Sigma\varphi$

6 Area of Application

The same mathematical concept can be expressed by different notations depending on the area it is used in (see Section 7 for more examples).

Imaginary Unit A mathematician uses the symbol i. In contrast, an electrical engineer uses j to avoid confusion with the symbol I for electric current. i and j are two alternative presentation for the symbol *imaginary unit*.

Inequality In mathematics $\neq$ is used, while computer scientists use $!=$ or $<>$.

Sometimes the same notation is used to present different mathematical concepts (see Section 8 for more examples):

Derivative In calculus f' stands for the first derivative of f but can mean any other entity different from f in other fields (cf. [RB97]).

More complex example include the *natural numbers*, which are defined and presented differently in various areas: A natural number (also called *counting number*) can mean either an element of the set $\{1, 2, 3, \ldots\}$ (the *positive integers*) or an element of the set $\{0, 1, 2, 3, \ldots\}$ (the *non-negative integers*). See below different notations and definitions for both concepts:

Positive Integers: $\{1, 2, 3, \ldots\}$

- Definition used in *number theory* (cf. [Wik08a]).
- cf. http://www.research.att.com/~njas/sequences/A000027
- Definitions used in Russia

Notation	Explanation	Source
$\mathbf{N}^+$, $\mathbf{N}^*$, $\mathbf{N}$	$\mathbf{N}^+$ (English), $\mathbf{N}^*$ (English), $\mathbf{N}$ (German, number theory: natural numbers are the numbers used for counting)	[DL08]
$\mathbf{N}^*$	The notation * is standard for non-zero or rather invertible elements, i.e. elements that can 'undo' the effect of combination with another given element.	[Wik08a]
$\mathbb{W}$, $\mathbb{P}$	Some authors who exclude zero from the naturals use the term whole numbers, denoted $\mathbb{W}$, for the set of non-negative integers. Others use the notation $\mathbb{P}$ for the positive integers.	[Wik08a]
$\mathbf{N}$	denotes natural numbers in Russia	Vyacheslav Zholudev
$\mathbf{N}_1$	???	unknown
$\mathbf{N}_+$	???	unknown

Non-negative Integers: $\{0, 1, 2, \ldots\}$

- Definition used in *mathematical logic*, *set theory*, and *computer science* (cf. [Wik08a]).
- cf. *Peano Axioms* [Wik08a] include 0 into the set of natural numbers (cf. [Wik08a]).
- cf. http://www.research.att.com/~njas/sequences/A001477
- cf. *Bourbaki* [Bou68]
- cf. *Halmos* [Hal74]

Notation	Explanation	Source
$\mathbf{N}$ or $\mathbb{N}$	Mathematicians use $\mathbf{N}$ (blackboard bold) or $\mathbb{N}$ (Unicode)	[Wik08a]
$\mathbf{N}_0$, $\mathbf{N}$	$\mathbf{N}_0$ (German, number theory), $\mathbf{N}$ (German, set theory), $\mathbf{N}$ (English)	[DL08]
$\mathbb{Z}^+$	$\mathbb{Z}^+$ signifies non-negative integers; is used in Russia to denote the set $\{0, 1, 2, \ldots\}$	[Wik08a], Vyacheslav Zholudev
ω	Set theorists often denote the set of all natural numbers by a lower-case Greek letter omega: ω. When this notation is used, zero is explicitly included as a natural number.	[Wik08a]

7 Same Concept But Different Notations

This section exemplifies mathematical object with the same meaning, which are presented differently depending on the mathematical subarea they are used in:

7.1 Basic mathematical symbols

I grateful acknowledge the use of examples from Wikipedia [Wik07b].

Mathematical Concept	Notations	context	reads
inequality	$\neq$ or $!=$ or $<>$	everywhere	is not equal to/ does not equal
inequality	$\leqslant$ or $<=$	order theory	is less than or equal to
inequality	$\geqslant$ or $>=$	order theory	is greater than or equal to
division	$\div$ or $/$	arithmetics	divide
multiplication	$\times$ or $\cdot$	arithmetics	times
material implication	$\Rightarrow$, $\rightarrow$, or $\supset$	prop. logic, Heyting algebra	implies; if ... then
material equivalence	$\Leftrightarrow$ or $\leftrightarrow$	prop. logic	if and only if; iff
logical negation	$\sim$ or $\neg$	prop. logic	not
definition	$\equiv$, $:=$, or $:\Leftrightarrow$	everywhere	is defined as
set builder notation	$\{:\}$ or $\{\|\}$	set theory	the set of ... such that
empty set	$\varnothing$ or $\{\}$	set theory	empty set
set-theoretic complement	$-$ or $\setminus$	set theory	minus, without
natural numbers	$\mathbb{N}$ or $\mathbf{N}$ or $\mathbb{N}_0$	numbers	N
integers	$\mathbb{Z}$ or $\mathbf{Z}$	numbers	Z
rational numbers	$\mathbb{Q}$ or $\mathbf{Q}$	numbers	Q
real numbers	$\mathbb{R}$ or $\mathbf{R}$	numbers	R
complex numbers	$\mathbb{C}$ or $\mathbf{C}$	numbers	C
real or complex numbers	$\mathbb{K}$ or $\mathbf{K}$	linear algebra	K
sum	$\sum$ or S; as well as $\sum_{k=1}^{n}$	arithmetics	summation over ...
derivative	$\cdot$ or $'$	calculus	... prime, derivative of
inner product	$\wedge$ or &	linear algebra	inner product of
exclusive or	$\oplus$ or $\veebar$	prop. logic, Boolean algebra	xor
logical truth	$\top$, T, 1	prop. logic, Boolean algebra	top
logical falsity	$\bot$, F, 0	prop. logic, Boolean algebra	bottom
disjoint union	$\cup$ or $+$	set Theory	the disjoint union

7.2 Hydrodynamics

In hydrodynamics the following appearances represent the same notation. People also call it “rotor (rotation)”, “curl” or “vorticity” depending on the appearance they choose: $\operatorname{rot} A$, $\nabla\times A$, $\operatorname{curl} A$, or $\nabla^{\perp} A$

8 Same Notation For Different Concept

This section exemplifies mathematical notations that present mathematical object with different meanings and provides the area they are most frequently used in.

8.1 Basic mathematical symbols

I grateful acknowledge the use of examples from Wikipedia [Wik07b].

N.	Math. Concept	Example	Math. Area	Notation Reads
$+$	addition	$1+3=4$	arithmetics	plus
$+$	disjoint union	$\{1,2\} + \{2\} = \{(1,0),(2,0),(2,1)\}$	set theory	the disjoint union of ... and ...
$+$	range	$\mathbb{Z}_+ = 1,2,3,\ldots$	numbers	
$-$	substraction	$3-1=2$	arithmetics	minus
$-$	negation	-1	arithmetics	negative, minus
				Continued on next page

Notation Collection – continued from previous page				
N.	**Math. Concept**	**Example**	**Math. Area**	**Notation Reads**
$-$	set-theoretic complement	$\{1,2,4,6,7\} - \{1,3,4\} = \{2,6,7\}$	set theory	minus, without; e.g. $A - B$ means 'the set that contains all the elements of A that are not in B' (but alternatively also \)
$\times$	multiplication	$2 \times 3 = 6$	arithmetics	times
$\times$	Cartesian product	$\{1,2\} \times \{3\} = \{(1,3),(2,3)\} \times$	set theory	reads e.g. $X \times Y$ means the set of all ordered pairs with the first element of each pair selected from X and the second element selected from Y'
$\times$	cross product	$(1,2,3) \times (4,5,6) = (-3,6,-3)$	vector algebra	cross, e.g. '$u \times v$ means the cross product of vectors u and v'
$\cdot$	multiplication	$2 \times 3 = 6$	arithmetics	times
$\cdot$	dot product		vector algebra	'dot', e.g. '$u \cdot v$ means the dot product of vectors u and v'
$\pm$	plus-minus	6 ± 3 means both $6+3$ and $6-3$	arithmetics	'plus or minus'
$\pm$	plus-minus	10 ± 2 or equivalently 10 ± 20 means the range from $10-2$ to $10+2$	measurement	'plus or minus'
$\mp$	minus-plus		is only used in arithmetics !!!	
$\sqrt{}$	square root	$\sqrt{x}$ means 'the positive number whose square is x'	real numbers	the principal square root of ...
$\sqrt{}$	square root		complex numbers	'the complex square root of ... '
$\mid ... \mid$	absolute value or modulus	$\mid x \mid$ means 'the distance along the real line (or across the complex plane) between x and zero'	numbers	absolute value (modulus) of ...
$\mid ... \mid$	Euclidean distance	$\mid x - y \mid$ means the Euclidean distance between x and y	geometry	Euclidean distance between ...; Euclidean norm of ...
$\mid ... \mid$	Determinant	$\mid A \mid$ means the determinant of the matrix A	Matrix theory	determinant of ...
$\mid$	divides	$a \mid b$ means a divides b	number theory	divides; a single vertical bar is used to denote divisibility
$\mid$	Conditional probability	$P(A \mid B)$ means a given b	probability	'Given' A single vertical bar is used to describe the probability of an event given another event happening
$\sim$	probability distribution	$X \sim D$, means the random variable X has the probability distribution D	statistics	has distribution
$\sim$	row equivalence	$A \sim B$ means that B can be generated by using a series of elementary row operations on A	matrix theory	is row equivalent to
$\wedge$	logical conjunction	The statement $A \wedge B$ is true if A and B are both true; else it is false	propositional logic	and; but also alternatively used: AND
$\wedge$	meet	For functions $A(x)$ and $B(x)$, $A(x) \wedge B(x)$ is used to mean $min(A(x), B(x))$	lattice theory	min
Continued on next page				

Notation Collection – continued from previous page				
N.	**Math. Concept**	**Example**	**Math. Area**	**Notation Reads**
$\vee$	logical disjunction	The statement $A \vee B$ is true if A or B (or both) are true; if both are false, the statement is false	propositional logic	or; but also alternatively used: OR
$\vee$	join	For functions $A(x)$ and $B(x)$, $A(x) \vee B(x)$ is used to mean $max(A(x), B(x))$	lattice theory	max
$()$	function application	$f(x)$ means the value of the function f at the element x	set theory	of
$()$	precedence grouping	Perform the operations inside the parentheses first: $(8/4)/2 = 2/2 = 1$, but $8/(4/2) = 8/2 = 4$	everywhere	parentheses
$\mathbb{C}$ or C	complex numbers		numbers	C
$\mathbb{C}$ or C	arbitrary constant	C can be any number, most likely unknown; usually occurs when calculating antiderivatives: if $f(x) = 6x^2 + 4x$, then $F(x) = 2x^3 + 2x^2 + C$, where $F'(x) = f(x)$	integral calculus	C
$\prod$	product	$\prod_{k=1}^{n} a_k$ means $a_1, a_2 \ldots a_n$	arithmetic	product over ...
$\prod$	Cartesian product	$\prod_{i=0}^{n} Y_i$ means the set of all $(n+1)$-tuples; the direct product of'	set theory	the Cartesian product of
$\int$	indefinite integral or antiderivative	$\int f(x)dx$ means a function whose derivative is f	calculus	indefinite integral of the antiderivative of
$\int$	definite integral	$\int f(x)_a^b dx$ means the signed area between the x-axis and the graph of the function f between $x = a$ and $x = b$	calculus	integral from ... to ... of ... with respect to ...
∇	gradient	$\nabla f(x_1, \ldots, x_n)$ is the vector of partial derivatives $\delta f/\delta x_1, \ldots, \delta f/\delta x_n)$	vector calculus	del, nabla, gradient of ...
∇	divergence	$\nabla \cdot \vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$	vector calculus	del dot, divergence of
∇	curl	$\nabla \times \vec{v} = \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right)\mathbf{i} + \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right)\mathbf{j} + \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\mathbf{k}$	vector calculus	curl of ...
$\perp$	perpendicular	$x \perp y$ means x is perpendicular to y; or more generally x is orthogonal to y	geometry	is perpendicular to ...
$\perp$	bottom element	$x = \perp$ means x is the smallest element e.g. $\forall x : x \wedge \perp = \perp$	lattice theory	the bottom element
$/$	quotient group	G/H means the quotient of group G modulo its subgroup H	group theory	mod
$/$	quotient set	$A/\sim$ means the set of all $\sim$ equivalence classes in A (If we define $\sim$ by $x \sim y \Leftrightarrow x - y \in \mathbb{Z}$, then $\mathbb{R}/\sim = \{\{x + n : n \in \mathbb{Z}\} : x \in (0, 1]\}$	set theory	mod
$\approx$	approximately equal	$x \approx y$ means x is approximately equal to y	everywhere	is approximately equal to ...
				Continued on next page

Notation Collection – continued from previous page				
N.	**Math. Concept**	**Example**	**Math. Area**	**Notation Reads**
$\approx$	isomorphism	$G \approx H$ means that group G is isomorphic to group H ($Q/1,1 \approx V$, where Q is the *quaternion group* and V is the *Klein four-group*)	group theory	is isomorphic to ...

8.2 Same Notation For Different Formulae

The formula $a_n = \frac{1}{2}n(n-1)$ describes different mathematical aspects: [Thi84]

1. The sum of the first $n-1$ natural numbers; e.g. for $n = 3$ $\sum_{k=1}^{n-1} k = \frac{1}{2}3(3-1) = 3$

2. The number of connecting passage between n points; e.g. for $n = 3$ points there are 3 connecting passages

3. The number of options to select 2 things simultaneously from a set of n things: e.g. given a set of 3 objects; there are three possibilities to select two objects at the same time

$O(n^2 + n)$ could present different terms. By common notational convention we know that the latter is most likely to hold. [Lan08]

1. O times $n^2 + n$

2. O (being a function) applied to $n^2 + n$

3. the set of all integer functions not growing faster than $n^2 + n$

8.3 Naming Conventions

Different Names For The Same Concept Russians add the names of Russian scientists to the names of theorems. For example, *Cauchy-Schwarz inequality* becomes the *Cauchy-Bunyakovski inequality*. In France, it is refereed to as *inégalité de Cauchy-Schwarz-Bunyakovskii*. [1]

Similar Names For Different Concepts In Contrast, adding a name to a theorem can change the meaning. For example, the *Cantor theorem*, *Cantor-Bernstein-Schroeder theorem*, or *Cantor-Bendixson theorem* are three different theorems.

Same Names For Diffferent Concepts The *Homogeneous equation* can mean:

1. An equations with the right-hand side equal to zero or

2. An equations with the left-hand side being a polynomial whose terms are monomials all having the same total degree.

[1] for a references to Cauchy-Schwarz-Bunyakovskii see Wikipedia at http://en.wikipedia.org/w/index.php?title=Cauchy%E2%80%93Schwarz_inequality&oldid=171567559 or Planetmath at http://planetmath.org/encyclopedia/CauchySchwarzInequality.html

Commonly Known Best Practices Polya [Pól73] presents several examples for best practices in mathematics:

- Use meaningful initials as symbols, such as a V for Volume or an r for radius.
- Notations are helpful when the order and the connection of the sign suggest the order and the connections of the objects, e.g. by using letters at the beginning of the alphabet (a, b, c) for given quantities or constants and letters at the end of the alphabet (x, y, z) for unknown quantities or variables.
- Denote objects that belong to the same category with letters of the same alphabet, i.e. Roman capitals (A, B, C) for points, small Roman letters as (a, b, c) for lines, Greek letters (α, β, γ) for angles.

9 Conclusion and Outlook

This paper summarizes various examples that illustrate the ambiguity of notations. In particular, we pointed out alternative notations that are used to present the same mathematical concept, i.e. mathematical content expression with the same meaning. In addition, we exemplified mathematical expression, which represent different mathematical concepts, but are presented with the same notation. Moreover, this work aimed at classifying alternative notations according to the context their are used in, in particular, by grouping them according to the dimensions *display style*, *level of expertise*, *language*, *individual styles*, and *area of application*.

The included examples are used to motivate and develop a sophisticated reader for mathematical documents that, based on the reader's context, adapts the mathematical notation of a document (cf. [pan]).

Acknowledgement I grateful acknowledge the use of examples from Wikipedia [Wik07b, Wik07a], and several publications on the topic [SW06, NW01, **?**, Thi84, RB97, DL08, MLUM06, Łuk67, Lan08]. I also want to thank the members of the KWARC group as well as all students of the General Computer Science Lecture for their valuable contributions.

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