Model Checking Timed Recursive CTL

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Temporal Logic with Recursion

proposal to enhance expressiveness of temporal logics: add recursion operator

e.g. CTL:

$$\varphi \quad ::= \quad q \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathtt{EX}\varphi \mid \mathtt{E}(\varphi \,\mathtt{U}\, \varphi) \mid \mathtt{EG}\varphi$$

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two types of formulas:

- propositional (φ): as usual interpreted in $2^{\mathcal{S}}$
- first-order (Φ): interpreted as function of type $2^{\mathcal{S}} \times \ldots \times 2^{\mathcal{S}} \to 2^{\mathcal{S}}$

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Proposition 1 ([B./Lange'20])

a) *RecCTL is more expressive than the modal μ-calculus.*

b) *Model checking RecCTL is* EXPTIME*-complete.*

Timed Recursive CTL

aim: extend high expressiveness with decidable model checking to verification of real-time systems

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Ex.: let $\Phi := \mathtt{rec}\, \mathcal{F}(x, y).(x \to y) \wedge \mathcal{F}(\mathtt{EF}^{\leq 3} x, \mathtt{EF}^{\leq 2} y)$, then

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TRCTL and Timed Automata

TRCTL interpreted over timed LTS, typically represented by timed automata (standard)

recursion formulas interpreted as least fixpoints in pointwise-ordered function space

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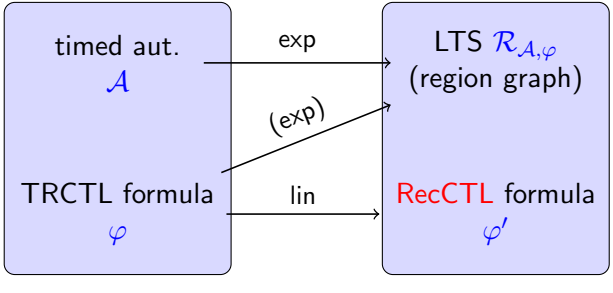
Theorem 1

Model checking TRCTL over timed automata is in 2EXPTIME.

PROOF: uses standard untiming region abstraction



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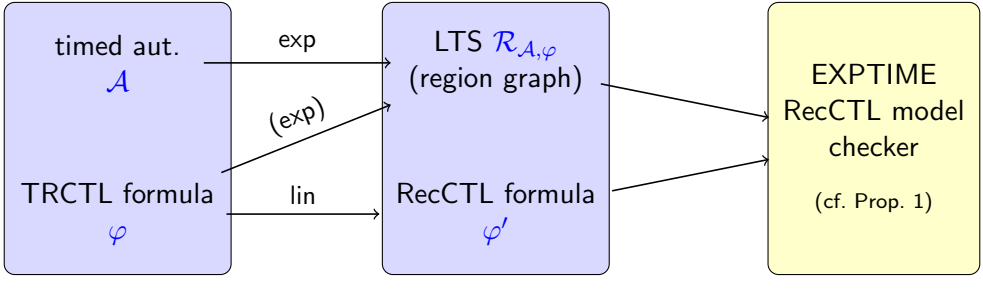
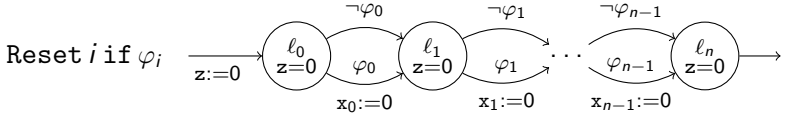


Manipulating Small Binary Counters in Timed Automata

Ex.: helpful "macro" timed automaton



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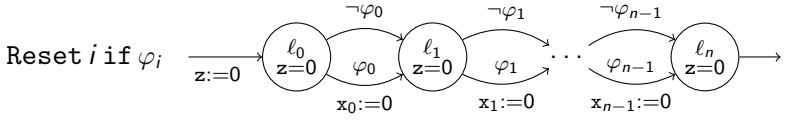
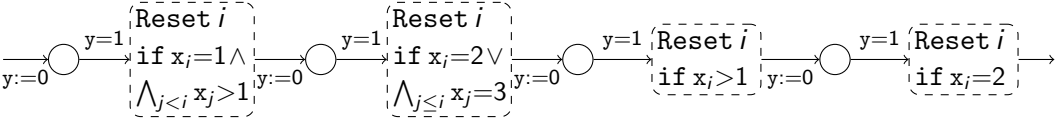


now what does



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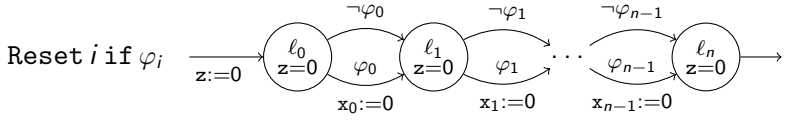

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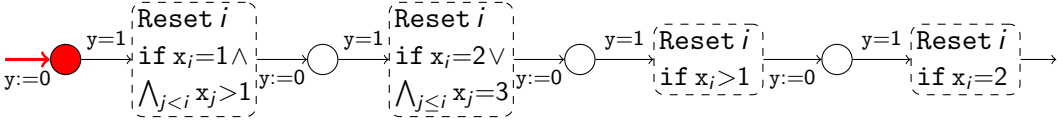

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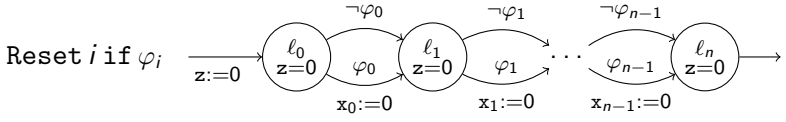

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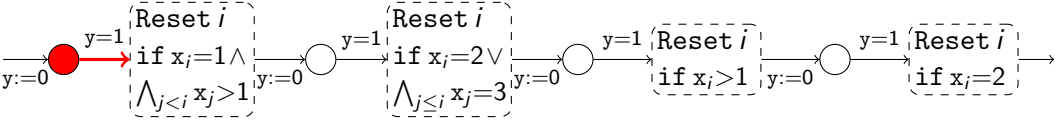

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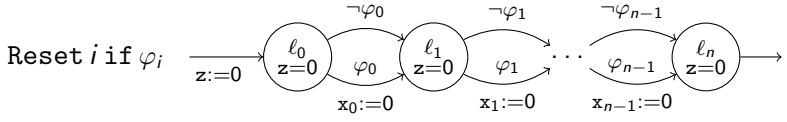

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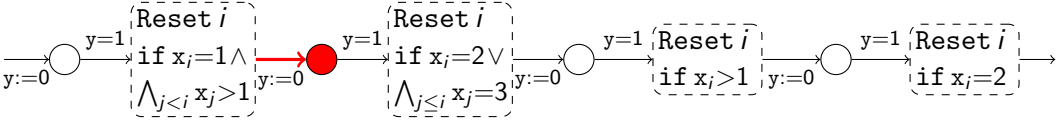

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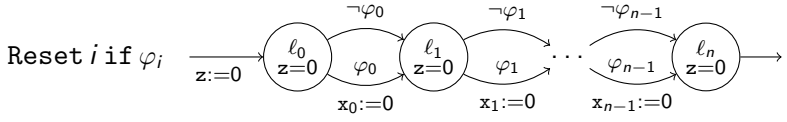

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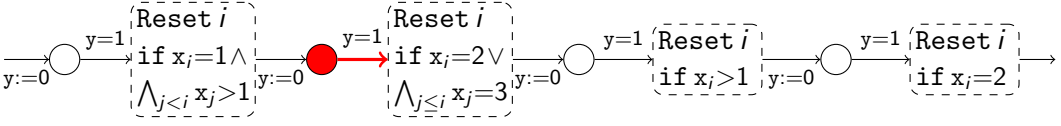

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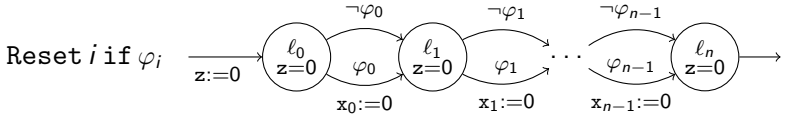

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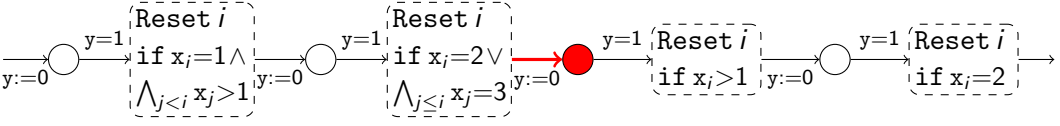

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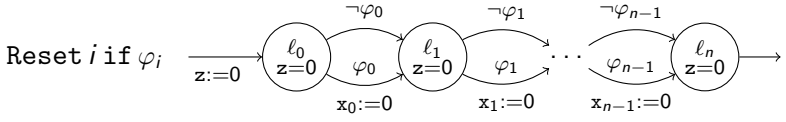

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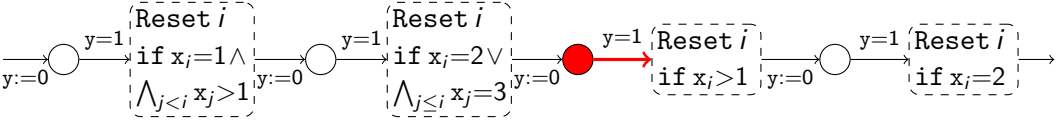

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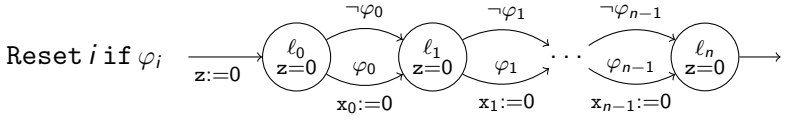

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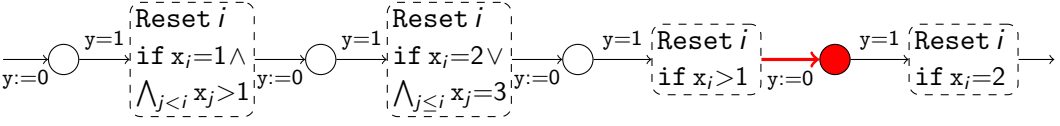

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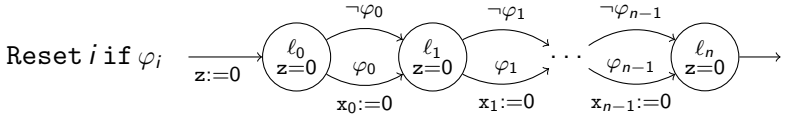

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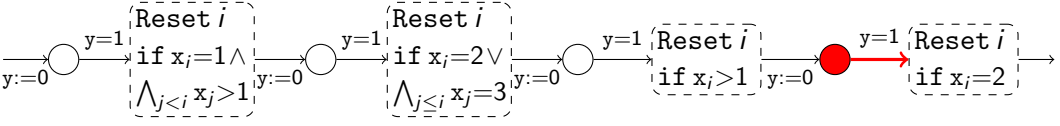

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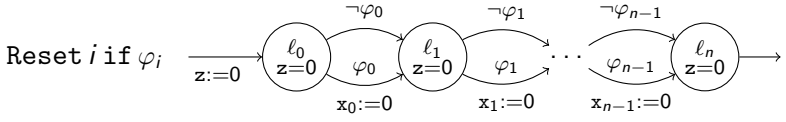

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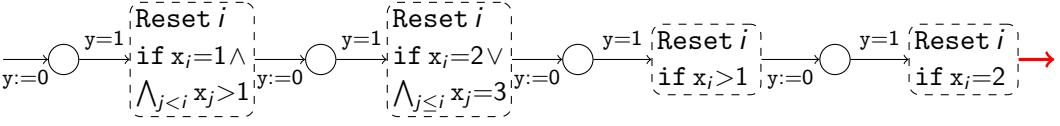

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Encoding Large Binary Counters in TRCTL Formulas

note:

- particular clock evaluations $\rightsquigarrow$ (small) counter for values in $[2^n]$
- set of small counter values naturally encodes (large) counter value in $[2^{2^n}]$

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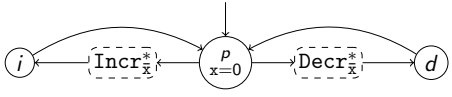
- particular clock evaluations $\rightsquigarrow$ (small) counter for values in $[2^n]$
- set of small counter values naturally encodes (large) counter value in $[2^{2^n}]$
- TRCTL first-order formulas can be used to manipulate arguments, representing large counter values

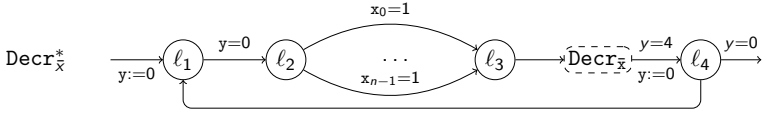
Lemma: if x encodes a large counter value, then

$$inc(x) = (x \wedge \mathtt{E}(\neg(p \vee i)\ \mathtt{U}^{>0}\ (p \wedge \neg x))) \vee (\neg x \wedge \mathtt{A}((\neg p \wedge \neg i)\,\mathtt{U}^{>0}\,(p \wedge x)))$$

encodes $x + 1$ in



with



The Lower Bound

Theorem 2

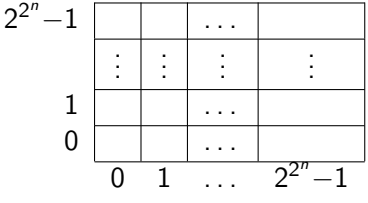
Model checking TRCTL over timed automata is 2EXPTIME*-hard.*

PROOF IDEA: given TM $\mathcal{M}$, $n \in \mathbb{N}$, construct TCTL formula of the form

$$\begin{aligned}(\,&\mathtt{rec}\; Chk(t,s).\ldots.Chk(dec(t),dec(s))\\ &\wedge Chk(dec(t),s)\\ &\wedge Chk(dec(t),inc(s))\ldots)(\mathtt{tt},\mathtt{ff})\end{aligned}$$

that checks for the existence of a tableau encoding an accepting run of $\mathcal{M}$

(uses characterisation of 2EXPTIME from [Chandra/Kozen/Stockmeyer'81])



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comments:

- lower bound shown for TA with non-constant number of clocks
- recent result: can be improved to one clock by relying more on higher-order mechanics (cf. [Laroussinie/Markey/Schnoebelen'04])
- lower bound holds for restricted sets of clock constraints (e.g. only $>$, only $=$), and already for expression complexity
- does model checking remain decidable over extensions of TA?