Assignment1 – Probability

Given: Apr 25   Due: May 2

Problem 1.1 (Simple Sample Spaces)

In many important situations including all problems treated in this course, the sample space, probability measure, domains, and random variables can be given in a simplified form, namely by:

- a list of random variable declarations $X_1, \ldots, X_n$, each consisting of
  - a name such as X
  - a finite domain such as $D_X = \{0, 1, 2, 3\}$
- a probability function $\Omega \to [0; 1]$ where $\Omega = D_{X_1} \times \ldots \times D_{X_n}$ such that $\Sigma_{e \in \Omega} P(e) = 1$

Define the corresponding probability space $\langle \Omega, Q \rangle$ and show that it satisfies the Kolmogorov axioms.
Define the random variables $Y_1, \ldots, Y_n$ induced by the respective X_i.

Problem 1.2 (Bayesian Rules)

Give the formulas and a one-sentence explanation of the following basic rules in Bayesian inference:

1. Bayes rule
2. Product rule
3. Chain rule
4. Marginalization
5. Normalization

Problem 1.3 (Basic Probability)

Let A, B, C be Boolean random variables, and let a, b, c denote the atomic events that A, B, C, respectively, are true. Which of the following equalities are always true? Justify each of your answers in one sentence.

1. $P(b) = P(a, b) + P(\neg a, b)$
2. $P(a) = P(a \mid b) + P(a \mid \neg b)$
3. $P(a, b) = P(a) \cdot P(b)$
4. $P(a, b \mid c) \cdot P(c) = P(c, a \mid b) \cdot P(b)$
5. $P(a \vee b) = P(a) + P(b)$
6. $P(a, \neg b) = (1 - P(b \mid a)) \cdot P(a)$

Problem 1.4 (Chained Production Elements)

An apparatus consists of six elements A, B, C, D, E, F. Assume the probabilities $P(b_X)$, that element X breaks down, are all stochastically independent, with $P(b_A) = 5\%$, $P(b_B) = 10\%$, $P(b_C) = 15\%$, $P(b_D) = 20\%$, $P(b_E) = 25\%$, and $P(b_F) = 30\%$.

Note: We deliberately differentiate between *not being operational* and *being broken*.

If an element breaks, it is not operational; if an element is not operational, either it or the linked element broke.

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1. Assume the apparatus works if and only if at least A and B are operational, C and D are operational, or E and F are operational. What is the probability the apparatus works?
2. Consider a different scenario, in which the elements A and C, D and F and B and E are pairwise linked; such that if either of them breaks down, then the linked element is not operational either. What is the probability that the apparatus works now?

Problem 1.5 (Probabilities in Python)

Complete the partial *implementation* of probabilities at `https://kwarc.info/teaching/AI/resources/AI2/probabilities/`