

Flyspeck in a Semantic Wiki – Collaborating on a Large Scale Formalization of the Kepler Conjecture

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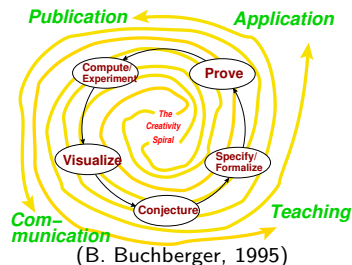
Overview

- The Flyspeck proof formalization project, and why it needs wiki support
- Requirements for a wiki to support Flyspeck
- Evaluation of Semantic MediaWiki and SWiM w. r. t. these requirements
- Further roadmap

Scientific Communication

Scientific communication means *collaborating on documents*

- Semantic markup languages support the workflow of structuring, annotating or reorganizing knowledge items
- Particularly common in the domain of mathematics (cf. preceding talk)
- SWiM is a semantic wiki for mathematical knowledge management
- We want to evaluate whether it will really support scientific knowledge engineering projects



The Kepler Conjecture



- 1611: Kepler conjectures that the max. density of packed unit spheres in 3D is $\pi/(3\sqrt{2})$
- 19???: Hilbert includes the conjecture in his 18th problem
- 1995: Hales proves the conjecture with massive computer usage:
 - 300 pages of human-readable text (theoretical foundations, proof outline)
 - thousands of lines of computer code (non-standard, not formalized for a proof assistant)
 - touches many areas of mathematics: plane, solid, spherical geometry, graph theory, hypermaps, single and multivariable calculus, plane and spherical trigonometry

Flyspeck

- 1995: Hales proves the Kepler conjecture
- Reviewers are 99% convinced but cannot verify the proof
- 2003: Hales initiates the **Flyspeck** project to fully formalize (“computerize”) the proof to a proof assistant’s level
- Expected duration: 20 man-years

Intermediate results of computerization:

- one fundamental algorithm in the computer code proven correct
- linear programming and global optimization code being investigated in two Ph. D. theses

Bulk of mathematical formalization remains to be done:

- elementary theories (e. g. spherical geometry) need to be defined (existing proof assistant libraries don’t contain them!)
- then the specific aspects of Kepler

Supporting Flyspeck in a Wiki

- Flyspeck makes an excellent use case for a semantic wiki
- Not only the highly formal proof is the goal, but also human-comprehensible description
- Large number of loosely coupled lemmas \Rightarrow many people can collaborate independently

Requirements for a wiki:

- human-readable presentation of descriptive text
- support for stepwisely computerizing human-readable informal text
- support for additional annotations, e. g. for discussing a formal definition, or for project management
- import/export interface to proof assistant(s), ultimately having the proof assistant integrated into the wiki (cf. MathWiki)

Scenario

- 1 Browse for to-dos
- 2 Download (with dependencies) into local proof checker
- 3 Browse wiki for help, discuss existing formalizations, refine informal annotations
- 4 Upload computerized proof, wiki checks it

To do		
Topic	Lemma	Discussion
Trigonometry	1.3	5 posts
Hypermaps	4.2	...

[Download computerization]
Type: Overview

1. Browse

references ↓

Lemma 1.3
 The cosine is an even function.
 The sine is an odd function.
 $\cos(-x) = \cos(x)$
 $\sin(-x) = -\sin(x)$
 [Download computerization]
 Type: Lemma, Topic: Trigonometry
 Proven: no (3 attempts)

2. Download

usesSymbol →

Cosine
 $\cos: \mathbb{R} \rightarrow \mathbb{R}, x \mapsto \dots$
 [Download computerization]
 Page type: Definition
 Topic: Trigonometry

Wiki Structure

- Knowledge base: one page contains one theory, symbol definition, lemma, or proof
- One discussion page per knowledge item, discuss issues related to the knowledge item there
- Theory browser: browse e. g. by topic (“spherical geometry”), or by logical dependency
- Editor: annotate and structure semi-formal texts, refactor definitions and theories
- Download (with dependencies), Upload (with check)
- Query interface:
 - “Which lemmas about composite regions need to be proved?”
 - “What lemmas are difficult to prove?”
 - “What do I need to understand the Jordan Curve Theorem?”
 - “What other lemmas could help me to prove this one?”

Evaluation of two Wikis


What is the right wiki to support Flyspeck?

- Data at our disposal:
 - the $\text{T}_{\text{E}}\text{X}$ sources of the Flyspeck outline
 - a Twelf computerization of lemmas from trigonometry
- Imported these into Semantic MediaWiki and SWiM
- Evaluated how much of the Flyspeck requirements these existing systems fulfilled

Semantic MediaWiki

- imported Twelf computerization via a custom upload page, enhanced it to improve navigation (symbols linked to their declaration, automatic categorization)
- one page for every computerized lemma, transcluded into one with additional comments and annotations
- Queries easy but not powerful enough (e. g. no negation):
[[Category:Unproven]] [[Category:Lemma]]
[[Category:Trigonometry]] [[written in::Twelf]]
- Ad hoc ontology development found useful for project management
- Importing existing OMDoc ontology for mathematical knowledge: could merely reuse vocabulary, no inference supported
- \LaTeX formulæ not semantically structured

Semantic MediaWiki



[Change](#) [my talk](#) [my preferences](#) [my watchlist](#) [my contributions](#) [log out](#)

article
discussion
edit
history
protect
delete
move
watch
refresh

Add your own math project to this wiki!

Flyspeck/lemma-1-11

< Flyspeck

Twelf

[edit]

```

lemma-1-11 : |- - one <= y
              -> |- y <= one
              -> |- sin(arccos y) == sqrt(one - sqr y).
    
```

LaTeX

[edit]

If $y \in [-1, 1]$, then $\sin(\arccos(y)) = \sqrt{1 - y^2}$.

Annotations

[edit]

This lemma was proven by Michael, using Isabelle. ([Proof object](#))

Facts about Flyspeck/lemma-1-11 ⓘ RDF feed ⓘ

Uses symbol [Flyspeck/lemma-1-11](#) + [Flyspeck/ bar-](#) + [Flyspeck/ til](#) + [Flyspeck/one](#) + [Flyspeck/ ltr](#) + [Flyspeck/sin](#) + [Flyspeck/arccos](#) + [Flyspeck/==](#) + [Flyspeck/sqrt](#) + [Flyspeck/-](#) + [and](#) + [Flyspeck/sqr](#) +

Written in [Twelf](#) + [Flyspeck](#)

Categories: [Flyspeck/other inverse trig functions](#) | [Lemma](#) | [Proven](#)

navigation

- [Main Page](#)
- [Current events](#)
- [Recent changes](#)
- [Random page](#)
- [All pages](#)
- [Help](#)

mathweb

- [Projects](#)

structures

- [Categories](#)
- [Properties](#)

search

toolbox

- [What links here](#)
- [Related changes](#)
- [Upload file](#)
- [Special pages](#)




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SWiM

- Manually converted some Twelf to OMDoc, SWiM's native math representation language (like OpenMath, but more expressive) (Automated conversion possible, too)
- SWiM supports OMDoc's ontology: types of mathematical knowledge items and their interrelations, e. g. "Proof proves Assertion"
- Structured discussion pages: infrastructure for issue tracking and resolving (work in progress)
- SPARQL inline queries:

```
SELECT ?l WHERE {
  ?l rdf:type odo:Lemma .
  ?l swrc:isAbout <Composite_Regions> .
  OPTIONAL {
    ?p rdf:type odo:Proof .
    ?p odo:proves ?l .
  }
  FILTER ( ! bound(?p) ) }
```

- Annotation not as straightforward as in Semantic MediaWiki
- Good RDF-based browsing, graph browser
- Reasoning powerful in principle (Pellet DL reasoner), but doesn't scale
- Semantically structured mathematical formulæ



User ▼

- Login
- Create Account
- Theme: [tundra] [soria]

Navigation ▼

- Recent Changes

Article

Discuss

Metadata

Context

Hierarchy

Edit

Annotate

History

Languages: [en] [de] [bg] [lt] [pl]

Lemma 1.11

Identifier: Lemma_1.11

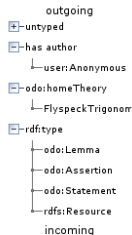
Types: `odo:Lemma` - `odo:Assertion` - `odo:NonconstitutiveStatement` - `odo:OHDocConcept` - `odo:Statement` - `rdfs:Resource`

LEMMA:

For the arccosine, the following property holds:

$$\forall y.(y \in [-1, 1] \Rightarrow \sin(\arccos(y)) = (\sqrt{1-y^2}))$$

References ▼



Related Work

- Combining computerized proofs and human-readable text: Isar, Mizar (but no web collaboration)
- Informal mathematical knowledge collections: Wikipedia, PlanetMath (no semantics)
- Wikis integrating proof assistants:
 - Logiweb: collaborative system, but hardly any browsing support. Interesting but idiosyncratic proof checker
 - ProofWiki prototype: Coq proof assistant inside MediaWiki; so far no semantics except Coq's, content either computerized or human-readable

Conclusion and Further Work

Continue enabling SWiM for Flyspeck (but Semantic MediaWiki was great for rapid prototyping!)

- Importing:** need to investigate existing $\text{L}^{\text{A}}\text{T}_{\text{E}}\text{X} \rightarrow \text{HTML} + \text{MathML}$ path, extend towards OMDoc, use SWiM's split-on-import
- Annotating:** need more flexibility; need refactoring assistance
- Browsing:** narrative book structure would be important for humans. Work aligning OMDoc and SALT ontologies in progress.
- Discussing:** integrating DILIGENT argumentation ontology with domain-specific extensions into SWiM
- Querying:** formula search not investigated, but solution exists. E. g. retrieve $\int_0^{\infty} f(y+z)dy$ by searching $\int_?^? f(x?z)dz$
- Different PAs:** For now focus on a single proof assistant (likely Isabelle), but logic translation would be nice
- Download:** computing dependencies: rules could work, but probably need a more sophisticated calculus
- Upload:** integrating theorem prover for checking uploaded data