Semantic Technologies for Mathematical eLearning*

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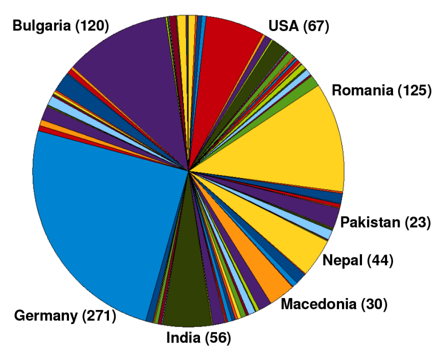
Abstract. With the globalisation in education, bridging cultural differences by making course material more accessible and adaptable to individual user needs becomes an important goal. In this paper we attack this goal for the field of mathematics where knowledge is abstract, highly structured, and extraordinary interlinked. Modern representation formats like our OMDoc format allow us to capture, model, relate, and represent mathematical learning objects and thus make them *context-aware* and *machine-adaptable* to the respective learning contexts. But to make mathematical knowledge accessible to learners of diverse cultural backgrounds we also need to model mathematical practice.
In this paper, we show that many practices of mathematical communities can already be modeled in OMDoc and outline extensions to support further ones. We have implemented a collection of services that allow applications to interpret and manage OMDoc and its practice representations. These services are integrated into our prototype eLearning platform to demonstrate how systems can improve the accessibility of mathematical eLearning materials.

1 Introduction



With the ever-increasing globalisation of higher education, learning institutions have to cope with culturally induced differences in prerequisite knowledge and learning practices. This is especially pronounced at Jacobs University Bremen with an international student body: 1100 students from 88 countries on April 8^{th}, 2008 [31].

Surprisingly, this also affects subjects like Mathematics and Computer Science that are often considered culture-independent. Even though most of our students are well prepared and possess good mathematical knowledge, a needs assessment study [1] shows mathematical discrepancies. Students of our one-year, introductory course on Computer Science (GenCS) reported that they had

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problems to get acquainted with the professor's notation systems, some had the feeling that the pace of the course was inappropriate and determined by the best students, some felt embarrassed to ask questions, while others did not face any problems and were able to balance out based on their previous education. Most students rate these discrepancies as problematic and believe that they can be associated with different educational and cultural backgrounds. In particular Romanian and Bulgarian students are very confident with their mathematical skills. Indian students are mostly well-educated in programming languages. Other nationalities struggle with the course. We assume that our students are capable of passing the course, but eventually give up when they are not able to map their previous mathematical background and practices to our course. In this situation, we want to augment lectures with online material that can be adapted to individual user needs.

We claim that the theory of *communities of practice* [17] can help us understand different mathematical practices and backgrounds and to eventually counteract pre-existing differences. According to Lave and Wenger [17], communities of practice (CoPs) are groups of people who *share an interest* in a particular domain — in our case the interest in the GenCS course. By interacting and collaborating around problems, solutions, and insights they *develop shared practices*, i.e. a common repertoire of resources consisting of experiences, stories, tools, and ways of addressing recurring problems. Even though mathematical practitioners seem to form a homogeneous, unified community and share the same practices all over the world, they actually form various sub-communities that differ in their preferred notations, basic mathematical assumptions, and motivating examples. We can observe these sub-communities among our GenCS students and see that exactly these communities are valuable for deepening knowledge and learning.

To allow our students to *access mathematical knowledge efficiently* in the online materials mentioned above, we explicate their *knowledge structure* and the *mathematical practices* and use these to support the students in interacting with the course materials.

For determining the knowledge structure we make use of the fact that mathematical knowledge is *abstract*, *highly structured*, and *extraordinary interlinked*. This allows us to more easily capture, model, relate, and represent *mathematical learning objects*. For the practices we use that mathematical communities often *interact via their mathematical knowledge artifacts*, such as theories or learning objects. We claim that their practices are *inscribed* into these artifacts. For example, mathematical authors *choose notations*, *make assumptions*, build on *different foundations* as well as results, and *choose typical examples* to illustrate their mathematical concepts [12].

Concretely we show in this paper, how to use our Open Mathematical Documents (OMDoc [14]) to represent mathematical learning objects as well as practices of mathematical communities. We illustrate which aspects of mathematical practices OMDoc supports and outline an extension of the format to further practices. We have implemented a collection of enabling technologies, which allow applications to interpret and manage OMDoc and its practice representations. Our enabling technologies are integrated into our prototype eLearning platform [24] to demonstrate how systems can improve the accessibility of mathematical eLearning materials.

2 Knowledge Representation for Mathematics

Mathematical Objects are what we talk and write about when we do mathematics: Rather simple objects like numbers, functions, triangles, matrices, and more complex ones such as vector spaces and infinite series. In order to provide automated services such as search or computation, we need to represent these objects in a machine-processable format, such as MathML [32] or OpenMath [27]. The former is a W3C recommendation for high-quality presentation of mathematical formulae on the Web, whereas the latter concentrates on the meaning of objects[1].

OpenMath Representation

```
<om:OMOBJ>
  <om:OMA>
    <om:OMS cd="combinat1"
             name="binomial" />
    <om:OMV name="n" />
    <om:OMV name="k" />
  </om:OMA>
</om:OMOBJ>
```

MathML Representation

```
<m:mrow>
  <m:mo>(</m:mo>
  <m:mfrac linethickness="0">
    <m:mi>n</m:mi>
    <m:mi>k</m:mi>
  </m:mfrac>
  <m:mo>)</m:mo>
</m:mrow>
```

Presentation

$\binom{n}{k}$

Fig. 1. OpenMath and MathML representation of the binomial coefficient.

Figure 1 provides the OpenMath and MathML representations of the number $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ of k-element subsets of a n-element set. The `OMS` element represents the "binomial coefficient" function, which (via `cd` and `name` attributes) points to a definition in a **content dictionary** (CD) [26]. CDs specify *commonly agreed* definitions of basic mathematical objects and allow machines to distinguish the meaning of included mathematical objects. Consequently, OpenMath expressions can be used by information retrieval or computation services while the MathML expression is used for display. A MathML-aware browsers presents the middle expression in Figure 1 as $\binom{n}{k}$.

The OMDoc format serves as *semantics-oriented representation format* and *ontology language* for *mathematical knowledge*. The format extends OpenMath and MathML with markup primitives for the structure and interrelations of mathematical objects expressed as **mathematical statements**, i.e. definitions, theorems, and proofs. We have already seen above that content dictionaries serve as an explicitly represented context for mathematical symbols, formulae and thus learning objects. The OMDoc format allows to represent CDs as OMDoc documents containing mathematical statements, but extends this functionality with a very expressive infrastructure for inter-CD relations that facilitate concept

[1] In fact MathML has a sub-language that is equivalent to OpenMath, but we concentrate on the presentational functionality of MathML for simplicity.

inheritance, parametric reuse, and multiple views on mathematical content. We claim that this **theory level** makes OMDoc an ideal representation format for **mathematical learning objects** (MLO), i.e. reusable, granular, highly structured, and semantically marked up fragments of varying size.

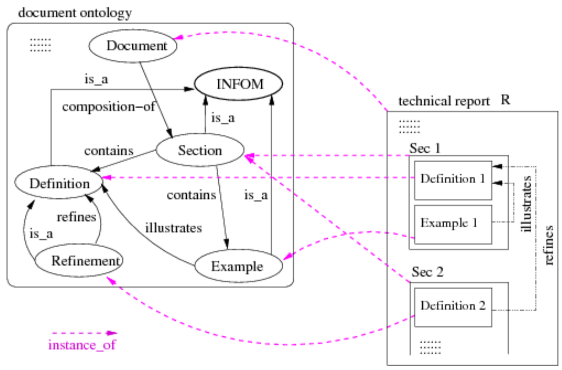
The OMDoc approach negates the intuition of existing eLearning approaches [4, 5, 18] that learning objects (LO) should be context independent. We believe that this aim is not only impossible to achieve, but also misleading. Authors are biased when creating LOs and always include subjective, context-dependent parts influenced by their didactic approaches or personal views. In mathematics, LOs also include the authors' individual and context-dependent practices such as their proving strategy, the choice of notations, or choice of typical examples. We believe that the context and practices of LOs should be represented explicitly so that machines can adapt them to the reader or learning goal. We call LOs with explicitly represented context-dependencies and practices "context-aware" to contrast them to the elusive "context-independent" ones. Thus, context-aware LOs allow to produce more accessible learning materials that are targeted to the individual needs and preference of the learner. For example, references to regional events or cultural aspects can motivate learners and allow them to more easily map new knowledge to prior experiences.

OMDoc allows to represent context-aware MLOs since it preserves the *logical*, *narrative*, and *social* contexts of MLOs and provides an infrastructure that interprets contextual information allowing for sophisticated semantic services [13]. For example, OMDoc supports a user-specific and context-aware selection and sequencing of LOs as well as their adaptive presentation that other non-semantic eLearning approaches can not offer [22].

3 Representing Practices in OMDoc

In the following, we detail how some of the practices above can be encoded in the OMDoc format. In this sense our paper can be seen as an instantiation of our earlier [13], where we state requirements for semantic representation formats for educational materials. We concentrate on three practices on different levels of the OMDoc format. The presentation of mathematical results (document level), the structuring and contextualisation of mathematical knowledge (theory level), and the choice of mathematical notations (object level). The approach exemplified with these examples can be applied to any practice. We analyse the mathematical objects affected by the practice and try to represent a functional core that is independent of the practice. Then we try to reify — i.e. turn into represented objects — the other factors or parameters in the practice in question. For instance for notation practices, the functional core is specified in form of the OpenMath and MathML formats, which define the representation of mathematical formulae. The practice factors are reified by representing the *choice of notation* for a semantic concepts and the *parameters* that can guide this choice as processable objects, see Section 3.2.

3.1 Representing Mathematical Documents



On the document level, OMDoc separates the narrative structure of mathematical documents from the content structure and thus makes it adaptable. The figure to the right[2] presents a technical report marked up using OMDoc's document ontology, which defines *narrative concepts*, such as section or document, content concepts, such example or definition, and their *interrelation*, e.g an example *illustrates* a definition and a document *is a composition of* sections. The narrative relations allow us to model the *didactic practice* of authors, i.e. their way of sequencing mathematical content. The content relations are the basis for automatic generations of these sequences. For example, we can compute a guided tour that summarises all mathematical preliminaries for a given concept (see [22] for details on the automatic selection and sequencing of documents).

3.2 Representing Notation Practices in OMDoc

```
<omdoc xmlns="http://omdoc.org/ns" ...>
  <notation> see Figure 3 </notation>
  <theory xml:id="MyTheory">
    <imports from="http://omdoc.org/combinat1.omdoc#binomial"/>
    <omtext xml:id="id2">
      <CMP>
        The binomial coefficients is the number of ways of choosing m objects from a
        collection of n distinct objects without regard to the order.
        We denote it by  see Figure 1 
      </CMP>
    </omtext>
  </theory>
</omdoc>
```

Fig. 2. OMDoc representation of an example document.

Figure 2 provides an example of a document represented in OMDoc. The OMDoc representation includes a `theory` element, which embeds a mathematical object represented in OpenMath. The `import` element specifies the required prior mathematical knowledge and is used analogously to operators in programming languages, which import required libraries and classes. In OMDoc, the `import` elements include all symbols from other theories that are used within the current theory, but have been defined and introduced in the imported theories. In the example, the `import` element of the theory `MyTheory` includes the symbol `binomial`, which is defined in the theory `combinat1`. Please note that

[2] The figure was borrowed from [25]

mathematical objects in OpenMath format can not be presented as OpenMath represents the meaning, but can not be used for display. These objects have to be converted to MathML. Consequently, we need to automatically process the author's notation practices, i.e. we need a mapping from OpenMath to a respective MathML representation.

In [15] we presented the extension of OMDoc towards the representation of mathematical notation practices to provide a flexible and context-aware conversion from OpenMath to MathML. We reified *notation preferences* of scientists into artifacts, that is *notation specifications*, which are applied onto the meaning of mathematical objects (represented in OpenMath) to generate their presentation (represented in MathML).

Figure 3 presents the OMDoc representation of a notation specification. The `prototype` pattern matches the OpenMath expression of the binomial coefficient in Figure 2. The `rendering` elements are applied to generate a concrete presentation for the symbol. The `context` attribute of the `rendering` element associates specific context parameters. In the example, the nationality of the respective notations are added. This allows to distinguish the German, Russian, and French notation of the binomial coefficient. Analogously, further context parameters such as the expertise level (novice, intermediate, expert) or area of application (mathematics, physics) can be added.

```
<notation xmlns:m="http://www.w3.org/1998/Math/MathML"
          xmlns:om="http://www.openmath.org/OpenMath">
  <prototype>
    <om:OMA>
      <om:OMS cd="combinat1"
              name="binomial" />
      <expr name="arg1"/>
      <expr name="arg2"/>
    </om:OMA>
  </prototype>
  <rendering context="language:Russian,ru">
    <m:msubsup>
    <m:mi>C</m:mi>
    <render name="arg1"/>
    <render name="arg2"/>
    </m:msubsup>
  </rendering>
  <rendering context="language:German,de">
    <m:mrow>
      <m:mo>(</m:mo>
      <m:mfrac linethickness="0">
        <render name="arg1"/>
        <render name="arg2"/>
      </m:mfrac>
      <m:mo>)</m:mo>
    </m:mrow>
  </rendering>
  <rendering context="language:French,fr">
    <m:msubsup>
      <m:mi>C</m:mi>
      <render name="arg2"/>
      <render name="arg1"/>
    </m:msubsup>
  </rendering>
</notation>
```

Fig. 3. An Example of a notation practice represented in OMDoc.

In order to select the appropriate presentation for a symbol, we proposed a context-aware conversion algorithm in [15]. First we collect all notation specification for a mathematical object, then we collect the user's context parameters for the conversion, and finally we select an appropriate `rendering` element which best fits to the current context and apply it to generate a presentation for the mathematical object. To provide a flexible and context-aware conversion algorithm, we provide various options to collect notation specifications as well as concrete context parameters.

Given the notation specification in Figure 3 and a concrete context parameter, the mathematical object in the OMDoc document in Figure 2 can be presented differently. For example, depending on the nationality selected by the user, the binomial coefficient is presented with its German ($\binom{n}{k}$), Russian (C_k^n), or French notation (C_n^k).

3.3 Representing Structure and Context of Math. Knowledge

To structure collections of learning objects and provide them with context OMDoc groups them into **theories** and links them via **theory morphisms**. This mechanism reifies a practice that long been relatively overt in mathematical documents, e.g. the Bourbaki development of mathematics that starts with set theory [3] and takes the mathematical practice of stating results with minimal preconditions to the extreme. OMDoc provides concrete markup for theory objects and extends the theoretically motivated accounts of inheritance and modularity in programming languages and mathematics to cover informal (but rigorous) mathematical practice (see [29] for the most recent theory, which will be incorporated into the upcoming version of OMDoc). Intuitively, a theory morphism is a mapping between theories that allows to "view" the source theory in terms of the target theory, if the mapping conserves truth. In the simplest case, theory morphisms model inheritance — the source theory can be viewed as an included part of the target theory — and thus allow to model the mathematical practice of modular/object-oriented development of knowledge in mathematics. For instance Figure 4 shows the inheritance graph of our GenCS course, and is used by students and the instructor for navigation and overview.

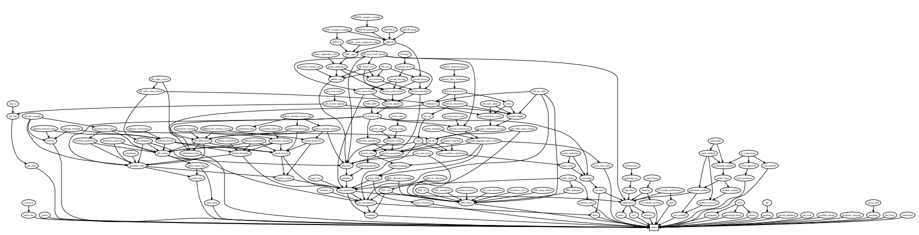


Fig. 4. The inheritance Graph of the GenCS course

But theory morphisms can also be used to model intra-mathematical differences in practices, e.g. differing choices of basic concepts. To gain and intuition, let consider an elementary example, the choice measuring temperature with the Kelvin, Celsius, and Fahrenheit scales. This example is suitable, since these scales make different defining assumptions — we model these as OMDoc `axiom` elements. For instance the Fahrenheit scale *defines* zero degrees to be the temperature of the coldest winter night Mr. Daniel Gabriel Fahrenheit ever experienced whereas the Celsius scale puts zero degrees a the freezing point of water, while the Kelvin scale puts it at the at hypothetical point, where all atoms cease motion (cf. Figure 5). The crucial observation is that (after suitable rescaling) all arrive at compatible consequences — which we model as OMDoc theorems. This allows us to establish the rescaling mappings as theory morphisms, since they are truth-preserving.

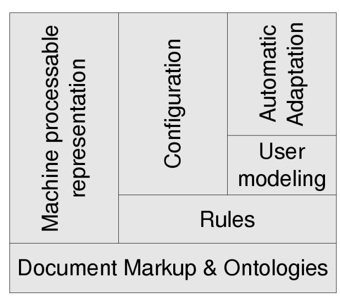
Theory	Temp. in Kelvin	Temp. in Celsius	Temp. in Fahrenheit
Signature	$^\circ K$	$^\circ C$	$^\circ F$
Axiom:	absolute zero at $0^\circ K$	Water freezes at $0^\circ C$	cold winter night: $0^\circ F$
Axiom:	$\delta(1^\circ K) = \delta(1^\circ C)$	Water boils at $100^\circ C$	domestic pig: $100^\circ F$
Theorem:	Water freezes at $271.3^\circ K$	domestic pig: $38^\circ C$	Water boils at $170^\circ C$
Theorem:	cold winter night: $240^\circ F$	absolute zero: $-271.3^\circ C$	abs. zero: $-460^\circ F$
Theory morphisms: $^\circ C \xrightarrow{+271.3^\circ} K$, $^\circ C \xrightarrow{-32/2^\circ} F$, and $^\circ F \xrightarrow{+240/2^\circ} F$			

Fig. 5. Three equivalent theories of temperatures

The important implication for eLearning is that the elaborate theory structure that was theoretically motivated originally can be utilised for adaptation and bridging of context differences. We can automatically re-contextualise learning objects. For instance we can move a LO from a Fahrenheit context to a Celsius context by translating it via the appropriate mapping above. This translation is safe, since we have established it to be a theory morphism earlier. Note that the re-contextualisation discussed here significantly surpasses the notation adaption discussed above as it is at the conceptual and content level. Another example of a service based on theory morphisms is to index *all* translations and thus make the virtual cloud of possible translations available to a formula search engine like our MathWebSearch service [16]. Our experiments show that even for an introductory course like GenCS supports about three dozen non-trivial theory views, while the theory views from subsequent course into GenCS go in the hundreds, since these courses are given by other instructors.

4 Implementation & Case Study

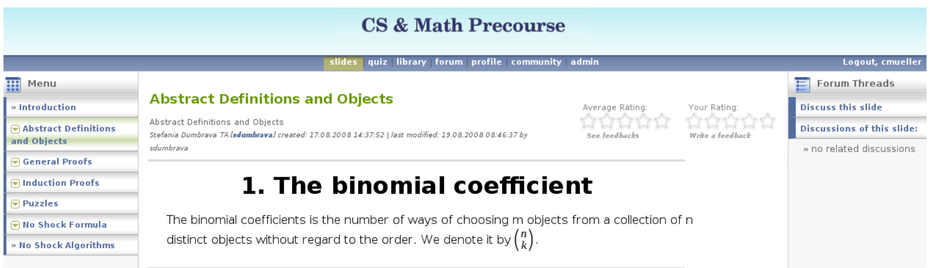
We propose an interactive environment, henceforth referred to as *active document environment* [21], which supports personalised interaction with mathematical content by adapting documents according to the user's practices and preferences. The figure to the right presents the constituents of active mathematical documents. The horizontal layers denote the preliminaries for the services in the vertical columns.



Document markup and ontologies provide a machine-processable representation of documents and allow us to distinguish the content, structure, and form of documents. In order to implement configurable documents, we extend the semantic representation layer with practice reifications, such as rules that express an authors choice of content, his way of sequencing content, or preferred presentation. For example, notation definitions are rules that define the rendering of mathematical concepts depending on the author's context. In order to provide automatic adaptation of documents, we need to model the user's behaviour, e.g. by applying user modeling techniques. For example, in [23] we extended our configuration of mathematical notation towards an adaptive framework that applies user modeling techniques to automatically adapt notations for the user.

To demonstrate our approach, we are implementing a proof-of-concept eLearning platform [24]. The implementation of the system is ongoing. In the following, we describe our envisioned prototype.

The sysem integrates the Java library JOMDoc [10] to convert lecture material in OMDoc+OpenMath into XHTML+MathML. During the import of OMDoc materials, the lecturer can specify his notation preferences. In the figure below, the German notation has been chosen. By integrating JOBAD [7] we can allow users to adapt the presented material on-the-fly, i.e. users can change notations and indicate their preferences while reading the material.



However, dynamic adaptation of material is not always in the intention of the lecturer, as he might wish to introduce specific notations and allow students to learn new ones. Consequently, systems should not simply overwrite the lecturer's notations, but rather add a hint for the user if his notation background differs with the presented symbols; see Figure 6 for two variants of the material above.

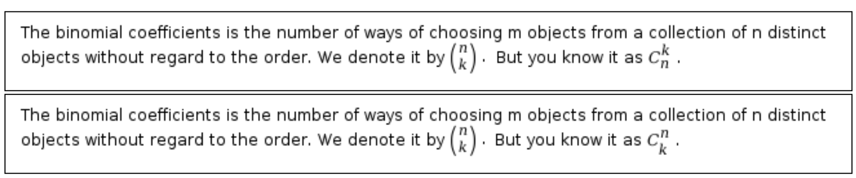


Fig. 6. User-specific Adaptation of Notations

5 Related Work

The ActiveMath system [19] integrates a framework for automatic course generation [30] based on hierarchical network planning, which produces personalised courses that adapt to the user's learning goals and competencies. Course are generated for different scenarios, such as the introduction of a learner to a previously unknown concept, the discovery of concepts, or the rehearsal of materials.

Content is represented in OMDoc, but the ActiveMath approach does not consider the full power of the format. Instead, the course generation is based on the annotation of didactic relations and properties. Practice-oriented adaptations along mathematical sound relations are not yet supported. However, the ActiveMath system offers a sophistiacted user modeling approach [20], which implements the third service layer of active documents, i.e. the system-driven adaptation of documents.

The educational knowledge repository ConneXions [2] is based on a corpus of semantic artifacts represented in CNXML [9], a *lightweight* XML markup language for educational content. CNXML *embeds* MathML as well as OpenMath for the representation of mathematical objects. It provides markup for the document level, but lacks markup of theories and theory dependencies. However, ConneXions provides "lenses" [11] that allow users to express their approval or rejection. These lenses are used to select appropriate content for a user according to his membership to a specific mathematical community.

[8] specifies strategies for interactive exercises. Strategies are procedures or procedural skills that help solving exercises and thus reflect mathematical practices, similar to the problem solving guidelines presented by Polyá [28]. Based on strategy specifications, [8] implement a web service, which is used by several mathematical eLearning applications allowing them to consider alternative strategies to provide more adaptive feedback and guidance.

The MathDox system [6] presents mathematical course material in form of interactive mathematical web pages. It integrates the MathDox Player and the previously mentioned exercise service [8] to implement accessible and living eLearning documents. However, although its XML-based document format embeds mathematical objects in OpenMath and MathML and provides a basic markup of the structure of documents, it is less suited for representing mathematical structures and practices as it is lacking OMDoc's theoretic foundations.

6 Conclusion & Outlook

In this paper we discussed how a modern, content-oriented document format (OMDoc) can be used to represent *context-aware* mathematical learning objects as well as practices of mathematical communities. We have shown how the context awareness of MLO representations together reified practices can be used to re-contextualise MLOs and adapt them to differing cultural backgrounds. The implementation of our prototype is work-in-progress and will be used to demonstrate how systems can improve the accessibility of eLearning materials.

Future work will address the reification of further practices in OMDoc, e.g. to support the automatic selection of typical examples and exercises for a given set of theories and user-specific context parameters. Moreover, we want to extend OMDoc to represent the social context of mathematical knowledge, i.e. information and relations that are capture during the user's interaction with mathematical knowledge such as tags, annotations, or discussions.

References

1. AAS. Need assessment report by the academic affairs committee of the undergraduate student government, 2008. private communication.
2. R.G. Baraniuk, C.S. Burrus, B.M. Hendricks, G.L. Henry, A.O. Hero III, D.H. Johnson, D.L. Jones, J. Kusuma, R.D. Nowak, J.E. Odegard, L.C. Potter, K. Ramchandran, R.J. Reedstrom, P. Schniter, I.W. Selesnick, D.B. Williams, and W.L. Wilson. Connexions: DSP education for a networked world. In *Proceedings of the Conference on Acoustics, Speech, and Signal Processing.*, *ICASSP* 4, 4144–4147. IEEE, 2002.
3. N. Bourbaki. *Theory of Sets*. Elements of Mathematics. Springer Verlag, 1968.
4. IEEE Learning Technology Standards Committee. Ieee standard for learning technology – data model for content to learning management system communication. `http://www.ieeeltsc.org/standards/1484-11-1-2004/`, 2005. seen May 2009.
5. IMS Global Learning Consortium. Learning resource metadata specification 2001. `http://www.imsglobal.org/metadata`, seen May 2009.
6. H. Cuypers, A. M. Cohen, J. W. Knopper, R. Verrijzer, and M. Spanbroek. MathDox, a system for interactive Mathematics. In J. Luca and E. R. Weippl, editors, *Proceedings of World Conference on Educational Multimedia, Hypermedia, and Telecommunications*, 5177–5182. AACE, 2008.
7. J. Giceva, C. Lange, and F. Rabe. Integrating web services into active mathematical documents. in press for Proceedings of the Conference on Intelligent Computer Mathematics.
8. B. Heeren, J. Jeuring, A. van Leeuwen, and A. Gerdes. Specifying Strategies for Exercises. In S. Autexier et al., editors, *Proceedings of the Conference on Intelligent Computer Mathematics*, *LNAI* 5144, 430–445. Springer Verlag, 2008.
9. B. Hendricks and A. Galvan. The Connexions Markup Language (CNXML). `http://cnx.org/aboutus/technology/cnxml/`, seen May 2009.
10. JOMDoc — a Java Library for OMDoc documents. `http://jomdoc.omdoc.org`, seen May 2009.
11. Christopher M. Kelty, C. Sidney Burrus, and Richard G. Baraniuk. Peer review anew: Three principles and a case study in postpublication quality assurance. *Special Issue on Educational Technology*, 96(6):1000–1011, 2008.
12. A. Kohlhase and M. Kohlhase. Communities of Practice in MKM: An Extensional Model. In J. Borwein and W. M. Farmer, editors, *Proceedings of the Conference on Mathematical Knowledge Management*, *LNAI* 4108, 179–193. Springer Verlag, 2006.
13. A. Kohlhase and M. Kohlhase. Semantic knowledge management for education. *Proceedings of the IEEE; Special Issue on Educational Technology*, 96(6):970–989, June 2008.
14. M. Kohlhase. OMDoc – *An open markup format for mathematical documents [Version 1.2]*. *LNAI* 4180. Springer Verlag, 2006.
15. M. Kohlhase, Christine Müller, and Florian Rabe. Notations for living mathematical documents. In S. Autexier et al., editors, *Proceedings of the Conference on Intelligent Computer Mathematics*, *LNAI* 5144, 504–519. Springer Verlag, 2008.
16. M. Kohlhase and I. Şucan. A search engine for mathematical formulae. In T. Ida et al., editors, *Proceedings of Artificial Intelligence and Symbolic Computation, AISC'2006*, *LNAI* 4120, 241–253. Springer Verlag, 2006.
17. J. Lave and E. Wenger. *Situated Learning: Legitimate Peripheral Participation*. Cambridge University Press, 1991.

18. Advanced Distributed Learning. Scorm: Sharable content object reference model. http://www.adlnet.gov/, 2000. seen May 2009.
19. E. Melis and J. Siekmann. Activemath: An intelligent tutoring system for mathematics. In L. Rutkowski et al., editors, *Proceedings of the International Conference on Artificial Intelligence and Soft Computing*, *LNAI* 3070, 91–101. Springer-Verlag, 2004.
20. E. Melis. User model description. DFKI Report, DFKI, 2001.
21. C. Müller. Active documents: personalised interactions with mathematical documents. submitted, 2009.
22. C. Müller. Specification and generation of variant documents. submitted, 2009.
23. C. Müller and M. Kohlhase. Context Aware Adaptation: A Case Study on Mathematical Notations.
24. C. Müller and M. Kohlhase. panta rhei. In A. Hinneburg, editor, *LWA Conference Proceedings*, 318–323. Martin-Luther-University Halle-Wittenberg, 2007.
25. N. Müller. An Ontology-Driven Management of Change. In A. Hanft, editor, *LWA Conference Proceedings*, 186–193. Universität Hildesheim, 2006.
26. OpenMath content dictionaries. http://www.openmath.org/cd/, seen May 2009.
27. OpenMath Home. http://www.openmath.org/, seen May 2009.
28. G. Pólya. *How to Solve it.* Princeton University Press, 1973.
29. F. Rabe. *Representing Logics and Logic Translations.* PhD thesis, Jacobs University Bremen, 2008.
30. C. Ullrich. *Pedagogically Founded Courseware Generation for Web-Based Learning*, *LNCS* 5260. Springer Verlag, 2008.
31. Jacobs University. Jacobs student body statistics, 2008. private communication.
32. W3C. Mathematical Markup Language Version 3.0. http://www.w3.org/TR/MathML3/, seen May 2009.